physics

PART ONE THIRD EDITION

PROFESSOR OF PHYSICS RENSSELAER POLYTECHNIC INSTITUTE

ROBERT RESNICK

PROFESSOR OF PHYSICS UNIVERSITY OF PITTSBURGH

DAVID HALLIDAY

JOHN WILEY & SONS, NEW YORK SANTA BARBARA LONDON SYDNEY TORONTO

In most problems in uniformly accelerated motion, two parameters are known and a third is sought. It is convenient, therefore, to obtain relations between any three of the four parameters. Equation 3-12 contains v_x , a_x , and t, but not x; Eq. 3-14 contains, x, v_x , and t but not a_x . To complete our system of equations we need two more relations, one containing x, a_x , and t but not v_x and another containing x, v_x , and a_x but not t. These are easily obtained by combining Eqs. 3-12 and 3-14.

Thus, if we substitute into Eq. 3-14 the value of v_x from Eq. 3-12, we thereby eliminate v_x and obtain

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2. ag{3-15}$$

When Eq. 3-12 is solved for t and this value for t is substituted into Eq. 3-14, we obtain

$$v_x^2 = v_{x0}^2 + 2a_x(x - x_0). (3-16)$$

Equations 3-12, 3-14, 3-15, and 3-16 (see Table 3-1) are the complete set of equations for motion along a straight line with constant acceleration.

Kinematic equations for straight line motion with constant acceleration The position x_0 and the velocity v_{x0} at the initial instant t=0 are the given initial conditions)

Number	Equation	3 V	Contains			
		X	V_x	a_x	t	
3-12	v = v + o t					
3-14	$v_x = v_{x0} + a_x t$	×	1	1	./	
3-15	$X = X_0 + \frac{1}{2}(V_{x0} + V_x)t$	1	1	×	1	
3-16	$X = X_0 + V_{x0}t + \frac{1}{2}a_xt^2$	1	×	1	V/	
	$v_{x^2} = v_{x0^2} + 2a_x(x - x_0)$./	1	٧,	V	

A special case of motion with constant acceleration is one in which the acceleration is zero, that is, $a_x = 0$. In this case the four equations in Table 3-1 reduce to the expected results $v_x = v_{x0}$ (the velocity does not change) and $x = x_0 + v_{x0}t$ (the displacement changes linearly with time).

The curve of Fig. 3-7b is a displacement-time graph for motion with constant acceleration; that is, it is a graph of Eq. 3-15 in which $x_0 = 0$. The slope of the tangent to the curve at time t equals the velocity v_x at that time. Notice that the slope increases continuously with time from v_{x0} at t=0. The rate of increase of this slope should give the acceleration a_x , which is constant in this case. The curve of Fig. 3-7b is a parabola since Eq. 3-15 is the equation for a parabola having slope v_{x0} at t = 0. We obtain, on successive differentiation of Eq. 3-15,

$$x = x_0 + v_{x0}t + \frac{1}{2}a_xt^2$$

$$dx/dt = v_{x0} + a_xt \quad \text{or} \quad v_x = v_{x0} + a_xt,$$

which gives the velocity v_x at time t (compare Eq. 3-12), and

$$dv_x/dt = a_x$$

the constant acceleration. The displacement-time graph for uniformly accelerated rectilinear motion will therefore always be parabolic.

You should not feel compelled to memorize relations such as those of Table 3-1. The important thing is to be able to follow the line of reasoning used to obtain the results. These relations will be recalled automatically after you have used them repeatedly to solve problems, partly as DIMENSIONS

EXAMPLE 4

CONSISTENCY OF