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## Uncertainty in the Yellow Change Interval

### Introduction

Up until the red light camera, it was the sole responsibility of a policeman to personally enforce the local ordinances prohibiting red light running. Rarely is there a problem when a policeman enforces the law. The good policeman understands that the driver has a certain amount of guess work when deciding what to do with a yellow light. So the policeman allows a driver to run a red light up to some small amount of time, somewhere around one second.

Then things changed. Red light cameras began enforcing the red light running laws with zero tolerance. As opposed to policemen, the cameras expect all drivers at all times in all circumstances and in all weather conditions to make stop and go decisions with infinite accuracy. So is the policeman giving the driver grace or is he subconsciously giving the driver the necessary tolerance as required by the underlying engineering?

### Engineering Requires Law Enforcement to Show Tolerance

Engineering practices require law enforcement to show tolerance because the yellow change interval is imprecise. The yellow change interval is calculated by a formula. Because the inputs to the formula are not precise, neither is the yellow change interval. The traffic engineer takes measurements of approach speed, perception/reaction time, deceleration rate, grade of road and intersection entry speed (for turning movements) and introduces them into the formula. Each of these measurements is not a fixed constant in the universe. Each value measures some human factor and as such has an uncertainty. The uncertainty in these “constants” propagate through the formula rendering an uncertain yellow change interval. By using the mathematical technique of *error propagation*, a technique common to physics experiments, we can compute the uncertainty of the yellow change interval. The engineer must share this uncertainty with law enforcement so that law enforcement grants drivers the proper leniency given the limits of engineering tolerance. The difference between legal and illegal when it comes to running a red light is not black and white. Engineering is never exact. Tolerance is routine for engineering. Yellow change intervals are not an exception.

## Straight-Through Movement Traffic

To make the math easier, we consider  $v_0$  as a true constant and ignore the grade of the road. We consider only the uncertainties in  $t_p$  and  $a$ .

Because the measured “constants” are not true constants and thus their measurements do not fall within a Gaussian distribution, the uncertainty in a computed result is the sum of the absolute values of the partial derivatives of the function with respect to each measured constant multiplied by the uncertainty of each measured constant.

<b>1</b>	<p style="text-align: center;">Yellow Change Interval Function for Straight-Through Movement<sup>1</sup></p> $Y \geq t_p + \frac{v_0}{2a}$
<b>2</b>	<p style="text-align: center;">Uncertainty in the Yellow Change Interval for Straight-Through Movement</p> $\Delta Y = \left  \frac{\partial Y}{\partial t_p} \Delta t_p \right  + \left  \frac{\partial Y}{\partial a} \Delta a \right $ <p>Where:</p> <p><math>\Delta t_p</math> is the uncertainty in the perception/reaction time.  <math>\Delta a</math> is the uncertainty in the deceleration rate of a vehicle.</p>
<b>3</b>	$\Delta Y =  \Delta t_p  + \left  -\frac{v_0}{2a^2} \Delta a \right $
<b>4</b>	$\Delta Y = \Delta t_p + \frac{v_0}{2a^2} \Delta a$

The yellow change interval formula is not your usual scientific function. Normally a scientist applies error propagation to a formula whose measured constants are truly constants. A constant has one and only one value in the universe. Discreet measurements of the constant result in a Gaussian distribution of values, the average of those values is the constant and the standard deviation is the constant's uncertainty. But the yellow change interval formula is different. What appears to be constants within the yellow change interval formula really is a range of equally valid values.

So instead of using the standard deviation for  $\Delta t_p$  and  $\Delta a$ , we use 1/2 of the range of their known values. For example we know that  $\Delta t_p$  to have a range from about 0.5 seconds to about 3.5 seconds (AASHTO's range)<sup>2</sup>. That range is about 3 seconds long which makes the uncertainty  $\Delta t_p$  to be approximately  $\pm 1.5$  seconds.

The following are some additional and important facts to keep in mind.

### **Perception/Reaction Time**

1. The yellow change interval formula treats perception/reaction time as a constant.
2. Perception-reaction time is often expressed in its two components: 1) perception time and 2) reaction time.
3. Perception time is the time a driver takes to perceive that the light turned yellow and to think about what action to take next.
4. Reaction time is the time it takes for a driver to move his foot to the brake pedal.
5. Reaction time is more of a constant than perception time.
6. Perception time is definitely not a constant. Though the ITE formula treats it as a constant, perception time is a complex formula.
7. Perception time is a function of the complexity of the intersection (AASHTO gauges the level of complexity in terms of "information bits"<sup>2</sup>), the distance the driver is from the intersection when the light turns yellow, the age of the driver, the alertness of the driver, the time of day, the weather, the vehicle's characteristics and perhaps many more variables.
8. Traffic engineers consider only measurements of the *combined* perception/reaction times. Engineers do not take measurements of perception time and reaction time separately.

9. Engineers restrict measurements of perception/reaction time to a subset of drivers. That subset is the set of drivers who stop. Engineers measure a perception/reaction time by clocking the time the light turned yellow to the time the car's brake lights come on.
10. Perception times applies to drivers who proceed into the intersection but engineers ignore these drivers. How long did it take for these drivers to make the decision to go? Engineers do not know but it is likely that limiting measurements to stopping perception times biases the results. Including perception times for proceeding drivers may result in perception-reaction times that are larger than what traffic engineers currently use. Larger perception-reaction times would increase the critical distance and thus would induce a type I dilemma zone given current yellow light intervals.
11. The quickest values of perception-reaction time are around 0.5 seconds. The slowest values of perception-reaction is about 3.5 seconds.
12. Traffic engineers lowball the perception-reaction time by systematically assuming a simple low-information intersection. ITE uses 1.0 second. Most traffic engineers adopt this value without studying the specifics of the intersection. The value for  $t_p$  however is really  $2.0 \pm 1.5$  seconds.

## **Deceleration**

1. The average deceleration of passenger vehicles is about  $10 \text{ ft/s}^2$ .
2. The deceleration of the best performance commercial drivers with an empty tractor-trailer truck on wet pavement<sup>3</sup> is about  $8.0 \text{ ft/s}^2$ .
3. The Transportation Research Board measured the maximum safe and comfortable deceleration of passenger vehicles is  $12.0 \text{ ft/s}^2$ .
4. Deceleration rates change in the rain or snow. Rain reduces the coefficient of friction on the pavement by 20 to 30 percent<sup>4</sup>. When a driver brakes on dry pavement at  $10.0 \text{ ft/s}^2$ , then pushes down his brake pedal to the same extent on wet pavement, he decelerates at  $8.0 \text{ ft/s}^2$ . Compensating for the rain requires an increase in the driver's perception/reaction time and depends on whether the vehicle's brakes/tires can exert a stronger opposing force in the first place.

5. We use  $\pm 1.2 \text{ ft/s}^2$  for the uncertainty in the deceleration rate in order to keep ITE's midpoint deceleration rate of  $10 \text{ ft/s}^2$ . We are underestimating the uncertainty here but it is not that important to be precise about the imprecision. It is important only to get a ballpark figure for the uncertainty in the yellow change interval.

## Plug in Numbers

<p><b>5</b></p>	<ul style="list-style-type: none"> <li>➤ The uncertainty in perception-reaction is <math>\pm 1.5</math> seconds.</li> <li>➤ The speed limit is 45 mph (66.2 ft/s).</li> <li>➤ The deceleration rate is <math>10 \text{ ft/s}^2</math>.</li> <li>➤ The uncertainty of the deceleration rate is about <math>\pm 2.0 \text{ ft/s}^2</math>.</li> <li>➤ From equation 4 the uncertainty in the yellow change interval is:</li> </ul> $\Delta Y = 1.5 + \frac{66.2}{2(100)} (2.0)$
<p><b>6</b></p>	<p>For a 45 mph speed limit, the uncertainty in the yellow light change interval for a driver who traverses unimpededly at the speed limit through the critical distance and into the intersection is:</p> $\Delta Y = \pm 2.2 \text{ seconds}$
<p><b>7</b></p>	<p>Using the middle of the range for perception/reaction time and deceleration, the proper way to express the yellow change interval for a 45 mph level road is:</p> $Y = 5.3 \pm 2.2 \text{ seconds}$ <p>Engineers use ITE values for <math>t_p</math> and <math>a</math> thus lowballing <math>Y</math> to 4.3 seconds.</p>

## Mandatory Directive from the Traffic Engineer to the Police Regarding Unimpeded Straight-Through Traffic Movement

“Because of the imprecision in the computation of the yellow change interval, you cannot legally ticket a driver unless the driver runs the red light by at least 2.3 seconds into the red. This tolerance is the engineering requirement for straight-through movement drivers who do not decelerate en route into the intersection. If you ticket these drivers, you are violating engineering practices and subsequently MUTCD standard 4D.26 (03).<sup>5</sup>

“A 5.3 second yellow interval seems long for a 45 mph level road. Yet commercial truck drivers, passenger vehicles travelling in bad weather and/or older drivers approaching a simple intersection may need more time than that. On the other hand, the 20 year-old driving a Maserati in good weather may need only 3.0 seconds. All these various types of vehicles and drivers are *allowed* on the road each requiring their own yellow change interval set by the laws of physics. Different strokes for different folks. Only a police officer at the scene can discern whether the red light runner is actually a violator. Engineers have to set the yellow light to something. By practice engineers omit telling law enforcement that they both lowball the duration for traffic efficiency reasons and that the duration really does require a tolerance.”

### Turning Traffic

To make the math easier, we consider only the uncertainties in  $t_p$ ,  $a$  and  $v_i$ .

<b>1</b>	<p>Yellow Change Interval Function for Turning Movement<sup>1,6</sup></p> $Y \geq \frac{v_0 \left( t_p + \frac{v_0}{2a} \right)}{\left( \frac{v_0 + v_i}{2} \right)}$ <p>Where <math>v_i</math> is the velocity of the vehicle at the stop bar.</p>
<b>2</b>	$Y \geq \frac{2v_0 \left( t_p + \frac{v_0}{2a} \right)}{v_0 + v_i}$

3

Uncertainty in the Yellow Change Interval for Turning Movement

$$\Delta Y = \left| \frac{\partial Y}{\partial t_p} \Delta t_p \right| + \left| \frac{\partial Y}{\partial a} \Delta a \right| + \left| \frac{\partial Y}{\partial v_i} \Delta v_i \right|$$

Where:

 $\Delta t_p$  is the uncertainty in the perception/reaction time. $\Delta a$  is the uncertainty in the deceleration rate of a vehicle. $\Delta v_i$  is the uncertainty in the intersection entry speed of a vehicle.

4

$$\Delta Y = \left| \frac{2v_0}{v_0 + v_i} \Delta t_p \right| + \left| \frac{v_0^2}{a^2(v_0 + v_i)} \Delta a \right| + \left| \frac{\partial Y}{\partial v_i} \Delta v_i \right|$$

5

Apply the derivative of the reciprocal law to the last term.

$$\frac{\partial Y}{\partial v_i} = \frac{\partial(1/f)}{\partial v_i} = -\frac{1}{f^2} \left( \frac{\partial(f)}{\partial v_i} \right)$$

6

$$f = \frac{v_0 + v_i}{2v_0 \left( t_p + \frac{v_0}{2a} \right)} = \frac{v_0}{2v_0 \left( t_p + \frac{v_0}{2a} \right)} + \frac{v_i}{2v_0 \left( t_p + \frac{v_0}{2a} \right)}$$

7

$$\frac{\partial(f)}{\partial v_i} = \frac{1}{2v_0 \left( t_p + \frac{v_0}{2a} \right)}$$

8

$$\Delta Y = \left| \frac{2v_0}{v_0 + v_i} \Delta t_p \right| + \left| \frac{v_0^2}{a^2(v_0 + v_i)} \Delta a \right| + \left| \left( \frac{2v_0 \left( t_p + \frac{v_0}{2a} \right)}{(v_0 + v_i)^2} \right) \Delta v_i \right|$$





9

- The uncertainty in perception-reaction is  $\pm 1.5$  seconds.
- The speed limit is 45 mph (66.2 ft/s).
- The intersection entry speed is 20 (29.4 ft/s).
- The range in the intersection entry speed is 10 mph to 35 mph.
- The uncertainty in the intersection entry speed is  $\pm 12.5$  mph (18.4 ft/s).
- The deceleration rate is 10 ft/s<sup>2</sup>.
- The uncertainty of the deceleration rate is about  $\pm 2.0$  ft/s<sup>2</sup>.
- Therefore from equation 8, the uncertainty in the yellow change interval is:

$$\Delta Y = \left| \frac{2(66.2)}{66.2 + 29.4} (1.5) \right| + \left| \frac{66.2^2}{(10)^2(66.2 + 29.4)} (2.0) \right| + \left| \left( \frac{2(66.2) \left( 2 + \frac{66.2}{2(10)} \right) \right)}{(66.2 + 29.4)^2} \right| (18.4) \right|$$

$$\Delta Y = |2.1| + |0.9| + |0.7|$$

10

$$\Delta Y = \pm 3.7 \text{ seconds}$$

11

Using the middle of the range for perception/reaction time and deceleration, the proper way to express the yellow change interval for a 45 mph level road for turning movement is:

$$Y = 7.4 \pm 3.7 \text{ seconds}$$

Because engineers' current practice is to misapply the straight-through movement formula to turning movements, yellow change interval are extremely short. States like Florida will set Y to 4.3 seconds. At its red light cameras intersections, Florida is currently increasing Y to 4.7 seconds. In Arizona, California, North Carolina and Virginia, engineers also plug in the wrong numbers into the wrong formula and set Y to 3.0 seconds.

## **Mandatory Directive from the Traffic Engineer to the Police Regarding Turning Traffic Movement**

“Because of the imprecision in the computation of the yellow change interval, you cannot legally ticket a driver unless the driver runs the red light by at least 3.4 seconds into the red. This tolerance is the engineering requirement for left and right turning drivers (U-turns even require more tolerance). If you ticket these drivers, you are violating engineering practices and subsequently MUTCD standard 4D.26 (03).<sup>5</sup>

“A 7.4 second yellow interval seems long for a 45 mph level road. Yet commercial truck drivers, passenger vehicles travelling in bad weather and/or older drivers approaching a simple intersection may need more time than that. On the other hand, the 20 year-old driving a Maserati in good weather may need only 4.0 seconds. All these various types of vehicles and drivers are *allowed* on the road each requiring their own yellow change interval set by the laws of physics. Different strokes for different folks. Only a police officer at the scene can discern whether the red light runner is actually a violator. Engineers have to set the yellow light to something. By practice engineers omit telling law enforcement that they both lowball the duration for traffic efficiency reasons and that the duration really does require a tolerance.”

## References

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- <sup>1</sup> Brian Ceccarelli, Joseph Shovlin, [\*Derivation of the Yellow Light Interval Formula\*](#), <http://redlightrobber.com> (October 2013).
- <sup>2</sup> American Association of State Highway and Transportation, [\*A Policy on Geometric Design of Highways and Streets\*](#), **2-40** (2011).
- <sup>3</sup>Transportation Research Board, [\*Review of Truck Characteristics as Factors in Roadway Design\*](#), *NCHRP Report 505*, Table 26, **48** (2003).
- <sup>4</sup> Hall, J.W., K.L. Smith, L. Titus-Glover, J.C. Wambold, T.J. Yager, and Z. Rado, [\*Guide for Pavement Friction\*](#), NCHRP Web-Only Document 108, Transportation Research Board of the National Academies, Washington D.C., **29**, (2009).
- <sup>5</sup> Federal Highway Administration, US Department of Transportation, [\*Manual of Uniform Traffic Control Devices\*](#), 2009 Revision 2, Section 4D.26, **485**, (2012).
- <sup>6</sup> Chiu Liu, et al., [\*Determination of Left-Turn Yellow Change and Red Clearance Interval\*](#), *Journal of Transportation Engineering*, Vol 128, Issue 5; equation 13, **454** (2002).