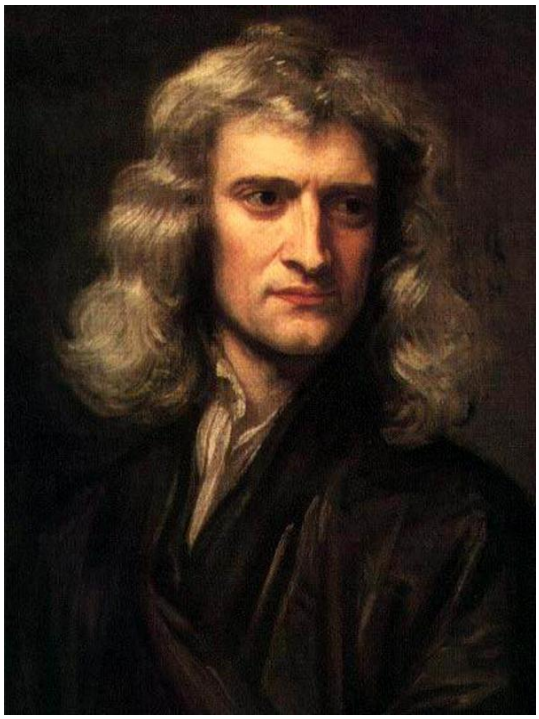
2012

Isaac Newton vs. Red Light Cameras



Derivation of the Yellow Light Interval Equation

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Setting the Length of the Yellow Light

Your Department of Transportation sets the length of the yellow light according to an equation published by the Institute of Transportation Engineers (ITE). The Institute of Transportation Engineers is an international organization. ITE was established in the United States in 1930. The yellow light equation has been in all editions of ITE's Traffic Engineering Handbook since 1965[1, 2, 3]. Most cities in the world apply this equation to their traffic lights.

ITE in turn, lifted its equation from the paper *The Problem of the Amber Signal Light in Traffic Flow*[4] published by Gazis, Herman and Maradudin in 1959. *The Problem of the Amber Signal Light in Traffic Flow* is the origin of the yellow light equation. Gazis, et. al. did their work on behalf of GM Research Laboratories.

ITE introduced the grade of the road into Gazis' equation in the *ITE Manual of Traffic Signal Design*, 1982.

The Physicist's Problem with the Equation

The ITE Yellow Light Change Interval Equation is not an equation of motion. While that statement gives physicists a heart attack, the statement doesn't mean anything to engineers. It takes a physicist about 20 seconds to look at the ITE Equation and say, "Oh my! That's forces drivers to run red lights. That's a Pandora's box." Apparently engineers can look at it for 50 years and still not see a problem.

ITE and the Institute Insurance of Highway Safety (IIHS) have published responses to the first editions of my paper. My first editions are dated before August 2010. I'll address their responses. ITE and IIHS said that I misunderstood the intent of the yellow light formula. Well, they were right. I did misunderstand then. But no longer. But knowing or not-knowing *its intent* does not affect

my conclusion. The equation is still wrong. The equation forces drivers to run red lights.

In my first editions, I could not imagine a professional engineering organization setting up a physics problem so incorrectly. So I mistakenly gave ITE the benefit of the doubt. I thought that ITE merely made a math goof. I though ITE accidently put the 2 in the denominator. So I wrote down the derivation of ITE's equation from first principles. I pointed out what I had assumed to be ITE's goof. I assumed that everyone would understand my conclusion, "Your equation is not an equation of motion. You can't do that." I ended my paper.

No engineer understood my conclusion. ITE does not understand what constitutes an equation of motion. ITE does not recognize that a bad equation applied reveals itself in systematic errors. ITE does not realize that its dilemma zones are the manifestations of the equation's systematic errors. And needless to say, ITE does not make the connection between the equation and the dilemma zones.

In July 2010, I discovered that ITE wants for that 2 to be in the denominator. I couldn't believe it. That 2 wasn't just a goof. Those guys really don't know physics. And as I have read more of ITE's papers, like *Determining Vehicle Signal Change and Clearance Intervals*, I have only confirmed that ITE does not know physics. ITE's method of determining the length of yellows for turn lanes[5] is a demonstration of work done in scientific darkness. Complete with "ifs" and "maybes", ITE's method would only be excusable had it been written before the Age of Enlightenment, the year 1687 and Newton's *Principia*.

The only thing I have changed in my paper since August 2010, is that I added an enumeration of the equation's deleterious effects on everyday drivers. As far as my mathematical derivation goes, that hasn't changed at all. It has been the same since day 1. In recent editions, I have added new discoveries. In September 2011, I discovered that the red light camera companies deliberately exploit the errors. The systematic placement of their cameras at specific intersections facing specific approaches shows explicit knowledge where the

equation's systematic errors are at their worst. *The red light camera industry knows*.

I'll address the defense that ITE, IIHS and engineering companies like Vanasse, Hangen and Brustlin give their "kinematic formula". They say their formula is based on a "sound application of the equations of motion" and "we have been using it for decades on our nation's highways" implying no need to revisit the formula. My rebuttal:

1. Though the formula does contain two separate valid equations of motion, the formula *unsoundly* combines them. The formula is an *unsound* application of equations of motion.

2. "We have been using the equation for decades." Using the equation for decades does not vindicate it. While about 98% of drivers do not experience one of the systematic errors at a given signal cycle[9], 2% do. 2% is a very large number of drivers. Millions each day. ITE forces 2 out of 100 drivers to play Russian roulette during each signal cycle. I believe that using the equation for decades explains the hundreds of thousands of crashes and deaths each year. It also explains the presence of the red light camera companies, vultures whose business is to eat the profit from the 2%.

3. You have to watch out for the motivations of ITE and engineering companies. Though I hate to jump to conclusions, there is financial motivation behind their common defense. ITE and engineering companies may defend themselves even when shown clear violations of physics. Their desire would be to stave off thousands of engineering malpractice lawsuits. If ITE admits physics errors in their widely-used yellow light duration standards, then ITE would be charged with malpractice. After all by definition, engineering is applied physics. ITE would be guilty of quackery. ITE could be charged with millions of injuries and deaths.

4. IIHS, the Insurance Institute of Highway Safety, promotes the use of red light cameras because IIHS makes a fortune off them. Red light camera tickets justify higher insurance premiums. Systematic errors from ITE mean profit for IIHS.

Gazis' Original Intent for the Yellow Light Equation

Denos Gazis invented the equation. But I cannot fault Denos Gazis because Gazis was explicit about the equation's intent and limitations. ITE is 100% to blame for taking Gazis out of context.

In Gazis' 1959 paper, Gazis plainly says that his equation is not a silver bullet. Gazis only intent was to make a horrible situation less horrible. Prior to 1959, traffic engineers set yellow light durations randomly. They'd set the yellow to 1 second at one intersection, 2 seconds at the next, and 6 seconds at the intersection across town. There was no rhyme or reason. Gazis wanted to introduce some consistent more safe method to it. Anything would be better than what was going on at that time.

So as a first step, Gazis took on one specific case, one specific case of traffic flow. It is the case where a driver is going straight, he is unimpeded by traffic in front of him, he can proceed to the intersection without decelerating, he can proceed at the maximum allowable speed (the speed limit), and he knows exactly how far back the distance $v^2/2a$ is from the intersection. For this one case, Gazis invented his formula.

That one case is the only case where the yellow light formula works.

But there is no case like this! This case does not exist in the real world. In the very least, do you know exactly where $v^2/2a$ is from the intersection?

ITE applies this equation to all cases. And it fails every time. That formula always force vehicles in everyday situations to run red lights. Mind you most people do get through intersections without incident. But about 2% will run the

red light because they will so happen to approach the intersection and be in the wrong spot just at the wrong moment where the equation does not work.

By applying and misapplying this formula, ITE has introduced systematic errors.

That is why people run red lights.

Systematic errors from the equation cause drivers to run red lights. Systematic errors are sure bets. Red light camera companies exist to exploit the sure thing.

The Contents of Pandora's Box

ITE's equation does not work for left, right and U turning drivers, for drivers who are approaching two-close-by intersections, or for any circumstance which causes a driver to slow down before entering the intersection. The equation means "go the speed limit when you enter the intersection." There is no extra allotment of time to compensate for slowing down. ITE, in a rash of pseudo-science[5], exacerbates the problem and sets turning yellows shorter than the equation, forcing even more drivers to run red lights. ITE does not understand that vehicles that are too close to the intersection to stop safely demand more time to slow down, not less. Going *slower* requires *more* time than the formula.

The definition of dilemma zone is now forthcoming. The phrase *dilemma zone* was coined by Denos Gazis. There are two types of dilemma zones[6]. A type I dilemma zone is a region on the road where if a vehicle is in it when the light turns yellow, the vehicle cannot stop safely nor proceed into the intersection without running a red light. No solution is possible.

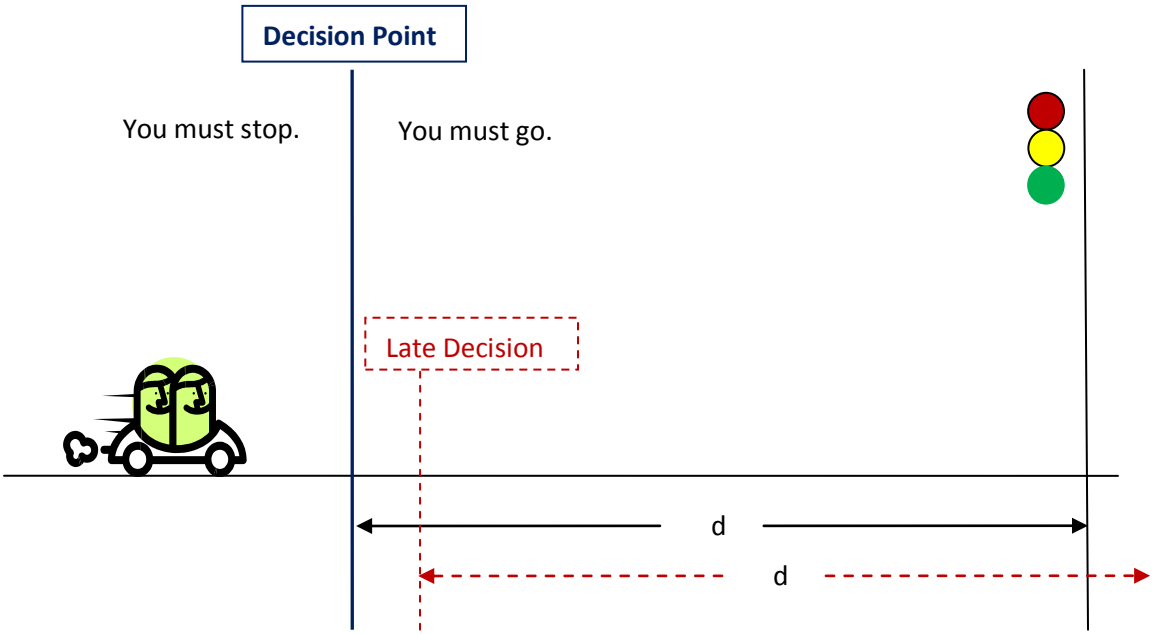
A type II dilemma zone is a zone of indecision where upon seeing the light turn yellow, the driver doesn't know whether to stop or go. A solution is physically possible, but he doesn't know what the solution is with certainty. He guesses. Half the time he runs a red light.

The Definition of a Yellow Light

When you approach an intersection and the light turns yellow, there is a point on the road where a decision to stop must be replaced by a decision to go.

The decision point is located at the safe stopping distance on the approach to the intersection. The safe stopping distance is also called the critical distance. The safe stopping distance is at $d = vt_p + v^2/2a$ where v = speed limit and a = deceleration of a car.

If we arrive at this **unmarked point** on the road at the **exact time the light turns yellow**, we can decide to stop or go and either decision is safe. If we decide to go, we will arrive at the intersection just as the light turns from yellow to red. If we decide to stop, we will stop at the intersection and the light will have already been red for several seconds.



Though this sounds reasonable, consider . . .

1. If we make a decision to stop too late, say just 2 feet after we pass the unmarked decision point, we will run a red light. We will stop 2 feet into the intersection because we still need the critical distance to stop. I am only stating laws of physics. The light is already red because the yellow light had already turned red long ago. ITE's yellow interval is half the length of time needed to stop.

   A late decision usually does not prevent us from stopping on time. We compensate by braking harder. However the later we decide, the harder it is to stop. There is also the problem of cars behind us. The harder we brake, the greater the likelihood they will rear-end us.

2. If we are farther away from the intersection than the critical distance when the light turns yellow, and we guess to go, then we enter the intersection **on a red** light while going the speed limit.

   When we see a yellow light, ITE's equation **biases** us to **go**. To go is logically our best option because we know **it is impossible to stop within the yellow interval.**

   Many times we are sincerely not sure what to do. In those cases we prefer acceleration because we want to be safe. We will chance a red light in favor of being safe. We speed up to make sure we clear the intersection before cross traffic gets a green. Speeding up is a natural outcome of the equation.[7] We also accelerate because we know that we cannot stop our car within the time the light is yellow. When acceleration is not enough, we will enter the intersection **on a red**.

It is true that we do often guess right. But also it is true we guess wrong. **Because the very nature of the equation presents us a guess**, the probability is > 0 that we will guess wrong at some point—perhaps a handful of times each year.

We will approach the intersection in just such a way that it is not clear what we should do. Since the equation biases us to beat-the-light, we will most likely go. It is a statistical certainty that we will run the red light at some point. Your DOT knows that is exactly what their dilemma zone does.

There is one solution.

1. To prevent people from running red lights, all we have to do is set the yellow interval to the stopping time. Make "yellow light means brake" the premise. If we do this simple thing, the light turning to yellow conveys a clear and simple meaning: By seeing the light turn yellow, we can now brake without penalty and our safety is guaranteed.

   No longer do we have to wage the debate, "Should I go or should I stop?" We can stop.

   Stopping on time may not always be possible, but at least we will glide thru the intersection ***on a yellow***. There will be no cross traffic because their light is still red. We are safe. Because we can stop comfortably, we don't have to worry about rear-enders. Cars behind us will not be surprised by a comfortable stop. We don't have to panic at red light camera intersections. We don't have to slam on the brakes to avoid a citation. We can rest in the fact that if we decide to try to stop, our try will not be penalized.

   Also, we can also decelerate. If we are turning or simply had to temporarily slow down for a car in front of us, we can decelerate and still enter the intersection.

Traffic Flow Preempts Safety

What was ITE thinking when they adopted their equation? Why did Denos Gazis entertain the idea that "yellow means go?" How do professional engineers justify their equation? Ask an engineer and he will tell you:

1. Because if we made the yellow light interval longer, we would increase traffic congestion. [More yellow means less green, and less green means fewer cars going through the intersection.]

2. If we make the yellow light the stopping time, that'll give people who do not stop too much time. They will treat the yellow as a green. [What the engineer does not understand is that the only alternative is to force the driver to treat the red as a green, which is what he does now.]

The story from traffic engineers is always the same. Engineers emphasize the need for cars to go over the need for cars to stop. ITE's yellow light equation is just one example. There are others:

1. Shorten left turn arrow yellows to 3.0 seconds as if all cars are going 20 mph--because most cars are in line waiting to turn left. But that's a big problem for cars that approach the intersection when there is no line waiting at the light. These cars approach at the speed limit. Did the laws of momentum suddenly change for objects in the left lane? Do you see a 20 mph speed limit sign in the left lane of a 45 mph road? For more information on the turning yellows, see Short Yellows and Turns.

2. Purposefully design an intersection forcing cars to run red lights. ITE actually recommends this. See page 412 in the Traffic Engineer Handbook, 6th edition, 4th paragraph from the bottom. ITE says that is okay for people to run red lights so long as DOTs provide enough all-red time so that the car can get across the intersection before cross-traffic gets a green. ITE doesn't think of the legal ramifications of making

people run red lights. ITE doesn't think of what happens to safety when they turn the all-red interval into a yellow. ITE forgets the purpose of the all-red interval.

Approach Speed vs. Speed Limit

At this point it would be good to mention the difference between approach speed and speed limit.

In the equations of traffic engineers, *v* is the approach speed. It is the speed which cars *approach* the intersection. More precisely speaking, the approach speed is the speed which cars enter the *safe stopping distance* segment of the road on their way to the intersection. That is original meaning of v is Denos Gazis equation[4].

The approach speed is not necessarily the speed limit.

In these papers, I simplify and say *v* is the speed limit. I do not make a distinction nor do I need to. My arguments apply to speed, regardless of speed's label. Also the Town of Cary is a flat landscape and so 95% of the time the approach speed is the speed limit.

I don't want to undercut the importance of the approach speed either. The topic of approach speed is important. It deserves its own paper. Refer to *Approach Speed vs. Speed Limit*.

In the end, though many cities violate the following equation, one must consider the approach speed for purposes of the setting the yellow light duration (not the all-red clearance interval) as:

$$v_{approach} \geq v_{speed\ limit}$$

The Erroneous Yellow Light Equation

The equation Departments of Transportation use to set the length of the yellow light is

$$\text{Yellow Interval} = t_p + v/(2a + 64.4g)$$

The equation expresses mathematically the premise that "yellow light means go." The premise comes from the paper *Determining Vehicle Signal Change and Clearance Intervals*.[5]

> "This formula for determining the length of the yellow change interval provides enough yellow time for a vehicle to travel, at its initial speed, over the distance it would take to stop at a comfortable average deceleration before entering the intersection."

Note that cars entering into the intersection are the ones that determine the yellow light interval. **Cars that go, not brake, determine how long the yellow light lasts. Yellow means go.**

The premise, "Yellow means go" is an error in physics. This is why:

- ✓ Red lights mean stop.
- ✓ Yellow lights exist only to serve red lights.
- ✓ Therefore yellow lights mean stop. The traffic engineer has a hard time with this. To accept the syllogism means he has to acknowledge a 100 year old error.

What is an Error in Physics?

I explain what a physics error is because I have found that most traffic engineers do not know what a physics error is. They do not understand that sound applications of science produce models that work all the time. Sound applications of science do not produce systematic errors like dilemma zones.

Traffic engineers claim that "the math is right."

The distance travelled by a car that stops divided by the speed of a car that goes = the yellow light time.

8 apples / 4 oranges = 2 pianos

The distance travelled by a car that stops is computed from the *velocity* of that car at the critical distance. But the average *velocity* of a car that intends to proceed through the intersection can be different. It is not necessarily the velocity of the car at the critical distance. The velocities are apples and oranges.

Traffic engineers do not see it. They see only 8/4 = 2. The "math is correct."

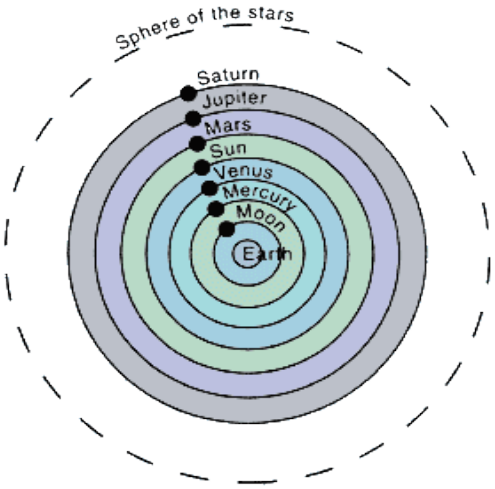
8 apples / 4 oranges = 2 pianos

is correct according to traffic engineers, albeit a flagrant violation of units, which violates the requirements of any physics equation, let alone an equation of motion. The "ITE Kinematic Formula" is therefore, not kinematic. It is not a valid equation at all. By using this equation, traffic engineers introduce systematic errors which force drivers to run red lights.

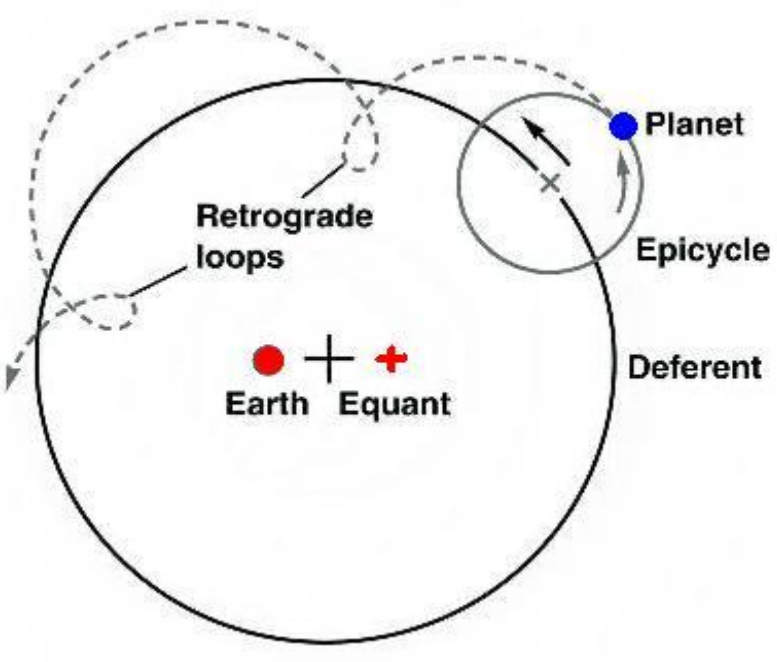
From history, I illustrate a different bad physics premise. This one is from the ancient Greek astronomer Ptolemy.

The Earth-Centric Universe

Ptolemy's premise was that the Earth was at the center of the Universe. ITE's premise is like Ptolemy's premise. Though well-intended, and it sort of works, it is still wrong:

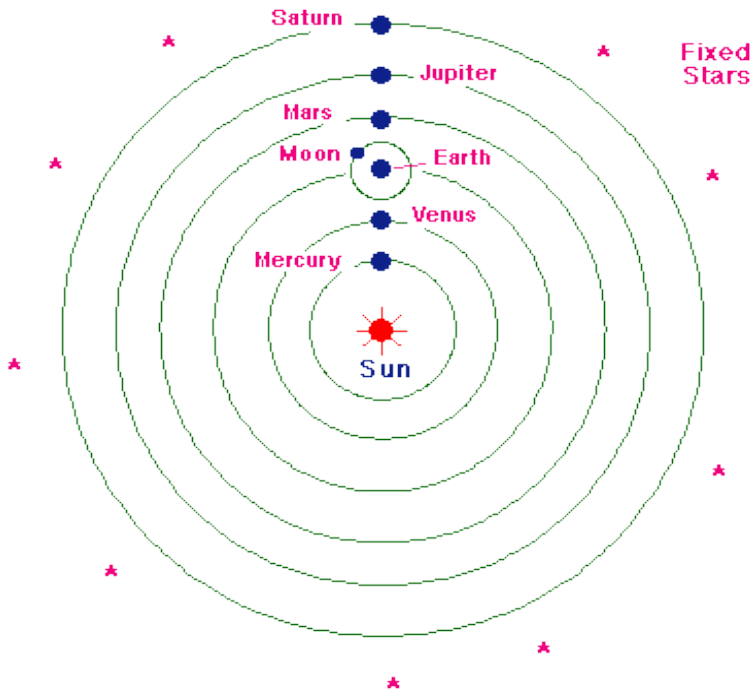


By using his model, Ptolemy could predict the motion of the planets. His predictions only worked to a point, because his math only worked to a point. His math couldn't explain planetary retrograde motion. His math had systemic errors because his math did not truly model the universe. Because the math had problems, Ptolemy understood that his premise was off. Had Ptolemy been a traffic engineer, he would have given each planet a red light camera ticket for illegal retrograde motion and left it at that. But Ptolemy was smarter. Ptolemy revised his premise. Instead of making the planets travel in perfect circles, he had them travel in Spirograph paths:



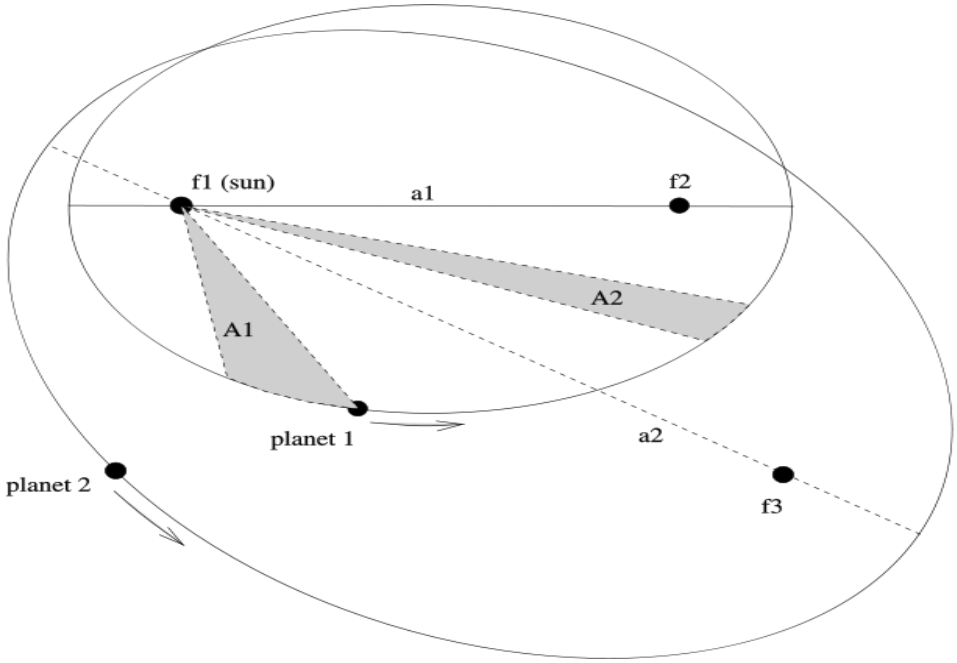
Each planet now danced in its own “epicycle” along its circular orbit. Ptolemy liked this better because it explained the observed motions of the planets better. The epicycle model held for centuries.

After 16 centuries, the astronomer Copernicus measured that Ptolemy’s math still didn’t describe actual observations. Ptolemy’s math worked great for Ptolemy’s premise, but reality exposed problems in Ptolemy’s math. By implication, that problem in the math meant a problem in the premise. Copernicus questioned the Earth-Centric Universe. So in order to make the math fit reality, Copernicus dissed Ptolemy’s premise and proposed that the Sun, not the Earth, is at the center of the Solar System:



Copernicus’ premise made things much better. By putting the Sun at the center, Copernicus could both explain the retrograde motion of the planets and describe the motions of the planets by using the equation of perfect circles.

As you know, Copernicus' math didn't quite hit the nail on the head either. His math also produced systematic errors. A century later, Kepler changed Copernicus's premise from circular orbits to elliptical orbits. The math of the ellipse, not the circle, perfectly predicts the planetary motions. To this day we use Kepler's premise and math equations:



Kepler's math models reality.

Back to red light cameras and yellow light equations.

Traffic engineers always claim the math behind the yellow light equation is right. But reality opposes the math. Measurements taken by the red light cameras observe that everyone in every city consistently runs red lights. That kind of observation means one of two things:

1. That either every one in every city is a bad driver, each one carelessly running red lights, or more likely . . .

2. The yellow light equation, which governs people running red lights, has systematic errors.

The reasons why 1 is false are:

- No one purposes to run red lights. Most people are not suicidal. No one looks at a red light and says, "Oh, I am going to intentionally run that light for the fun of it."

- One cannot believe that the population of an entire city consists of all bad drivers. The red light cameras have measured that 130,000 people in Cary are red light runners. That's everybody in town.

Thousands of cars running red lights is the tell-tale sign of a bad premise in action. These cars run red lights because the math of ITE's yellow light equation forces upon drivers a false reality which no one can obey.

Accepting ITE's Premise

If you accept ITE's premise, "yellow light means go," then you believe

1. That no one ever needs to decelerate and enter the intersection.
2. That no one ever turns left.
3. That no one ever turns right.
4. That no one has to tap his brakes for cars pulling into the road from business exits.
5. That no one has to tap his brakes for cars for any reason.
6. That no one has ever had to slow down for the next signal light on the far side of the signalized intersection.
7. That stopping and going, even though your life depends on it, should be a guess.
8. That it is okay for people to cause accidents because the equation offers no event which the driver can use to guarantee his safety.
9. That it is okay for the yellow light interval to be half that required to stop your car, despite that inducing a bias to go instead of stop.

10. That it is okay for the yellow light interval to be half that required to stop your car, despite that causing rear-end collisions.
11. That it is okay for the yellow light interval to be time required to traverse the safe stopping distance at no less than the speed limit, even though turning drivers slow down under the speed limit before entering the intersection, thus forcing them to run red lights.
12. That it is okay for the yellow light interval to be time required to traverse the safe stopping distance at no less than the speed limit, even though drivers may have to slow down under the speed limit to avoid hitting cars waiting at the next traffic light or pulling out of business exits, before entering the intersection, thus forcing them to run red lights.
13. That drivers who beat-the-light intentionally want to run red lights.
14. That it is okay to be penalized for braking when seeing a light turn yellow.
15. That it is okay to encourage full-speed T-Bone crashes.
16. That red light camera programs are a great way to make money, since the equation induces a guess and a bias which stack the deck in favor of the red light camera company.
17. That everyone in the Town of Cary is a felon because the Town of Cary has issued 130,000 tickets—equal to the population of Cary.
18. That it is okay to disregard places like Georgia who found that adding 1 second to all ITE's yellow light intervals reduced the red light runners by 80%. That forced the red light camera companies to pull out. [By increasing the yellow interval by 1 second, Georgia gets closer to the value Newton's Laws of Motion dictates. If Georgia increases the yellow time to what Newton's Law requires for stopping cars, Georgia will see their 80% decrease go to 99.9%.]

Accepting the Correct Premise

If you accept the correct premise, "yellow light means brake," then you believe

1. That traffic control devices should have a clear and simple meaning.
2. That seeing a light turn yellow should guarantee your safety.
3. That you have the time to decelerate to make a turn.
4. That you have the time to decelerate for obstacles in your path.
5. That you should never get penalized for braking.
6. That cars never need to rear-end you.

7. That skidding into the intersection on a yellow is better than on a red.
8. That running full speed into cross-traffic never has to happen.
9. That red light camera programs should never exist, for the only people running red lights would be the occasional drunk, and there's no profit in that.

North Carolina Law

ITE's equation appears in section 5, page 19 of the NCDOT Intelligent Transportation and Signal Systems Unit *Design Manual*. It is Std 5.2.2, Sheet 4 of 4.

Cary Town Charter 8.15 (prior to 2/2011) and N. C. Session Law 2001-286 state that the duration of the straight-thru yellow interval must equal or exceed the yellow interval from this equation. The newer Cary Town Charter App 2.8 and N. C. Session Law 2004-141 state the same thing indirectly. These newer versions of the law say that the traffic signal plans must be signed and sealed. The requirements of a signed and sealed signal plan require adherence[8] to the same NCDOT yellow light duration standard. Many of North Carolina's yellow intervals *do not equal or exceed* the yellow intervals from this equation. Those red light cameras, by City Charter and State Law, are explicitly illegal.

The Town of Cary ignores this yellow light equation when it shorts its independently phased left turn yellows. The Town of Cary opts to use a different NCDOT "standard," a standard which allows Cary to give a 45 mph car the braking distance of a 22.3 mph car. This standard originally comes from ITE[5].

Many yellow intervals do abide by ITE's equation. But then they either fall short 2 to 3 seconds as required by the intermittent need for cars to decelerate and enter the intersection, or that Cary fails to mark the safe stopping distance line as required by the equation.

Derivation of the Yellow Light Equation

The Instigator

After I got flashed by a red light camera, I discovered that my intersection's (Cary Towne Blvd. at Convention) yellow light interval did not meet the minimum required by ITE's equation. It turned out that the NCDOT made a mistake drafting the signal plan. The posted speed limit has always been 45 mph, but the signal plan said the road was a 35 mph road. The plan has been wrong for 21 years. As traffic engineers say, "What is on the ground is not what is in the plan." NCDOT had set yellow light interval for a 35 mph road. They set the yellow interval ½ second short. 6 months after Cary convicted me, on March 19, 2011 they increased the yellow light interval to that for a 45 mph road. Red light runners decreased by 80%. Cary did not refund anyone's money. From this one light, Cary illegally stole $427,950 according to their own Charter. Just for speculation, this intersection was the first where the Town of Cary installed a red light camera.

The same thing happened to Susan Sharma at a different intersection. She ran the red light at High House Road at Prestonwood in September 2006. That intersection's yellow light was also shorter than the minimum requirement. Cary convicted her anyway. One month later Cary increased the yellow light interval to that mandated by their Charter. Cary did not admit the problem. Cary did not refund her money. Cary stole $ 299,350.00 from this light.

To this day, Cary refuses to admit it.

Grail Quest

The legality of the red light cameras is built upon the validity of ITE's equation. Even though I already knew *my* red light camera was illegal, I still wanted to understand the equation. For me to complain about a yellow light interval

without understanding the principles the NCDOT uses to set them would mean my embarrassment in front of the judge. I want to be prepared.

I searched the internet for a derivation of ITE's equation but found none. I found the North Carolina Statutes, the Town of Cary Ordinances, the NCDOT Signals Manuals, the ITE Handbook and the Manual for Uniform Traffic Control Devices. But I could not find the derivation of the yellow light interval equation. Engineering books have the equation in them. Denos Gazis' original paper has the equation in it. But no book or white paper has a complete derivation from first principles. There is no book showing where the implicit $v^2/2a$ comes from, nor how the Gg term got there. Every DOT spec and engineering book adopts ITE's equation without inspection. Even ITE doesn't bother showing a derivation nor does ITE question its sources.

I have a B.S. in physics from the University of Arizona. I derived the equation and did so successfully. I know exactly where everything comes from and why. The problem is a simple classical mechanics problem. Any freshman physics student could do it. Yet the equation I derived was not ITE's equation.

Did I make a mistake? No I didn't. It turns out I could not derive ITE's equation because ITE's equation cannot be derived. That's because ITE's equation is *not an equation of motion*. *The equation does not describe the properties of a specific vehicle.* Half the time the equation describes the distance required by a stopping vehicle. Half the time the equation describes the time needed for a proceeding vehicle going the speed limit. When the equation describes one, it excludes the other. And then there is the problem that the equation does not at all handle decelerating vehicles that must enter the intersection. The problem is that equation does not handle a single vehicle completely.

Stopping Distance Equation

The way I discovered how ITE arrived at their equation was an accident. I stumbled upon the stopping distance equation. The stopping distance has that extra "2" in the denominator just like the ITE's yellow interval equation. The stopping distance does come from Newton's Laws of Motion. It includes the amount of distance a car travels during the perception interval and the amount of time a car takes to brake. Here's the equation, and it is correct:

$$S = t_p v_o + v_o^2/2(a_b + 32.2g)$$

The stopping distance equation has two parts.

1. $S_p = t_p v_0$ = perception distance

2. $S_b = v_0^2/2(a_b + 32.2g)$ = braking distance

Traffic engineers first ask the question: *What distance does the car travel from when the driver first sees the light turn yellow to where the driver comes to a stop at the intersection?* This first question is a consequence of the bad premise. That bad premise predates even Denos Gazis. Traffic engineers are interested in the distance it takes a car to stop, not how long it takes a car to stop. Traffic engineers are not interesting in how long it takes a car to stop, but rather for a car to proceed through this braking distance. Yellow light means go. The bad premise at work.

Stopping Distance Derivation

Here's the derivation of the stopping distance equation. Traffic engineers get this equation right. It is important for you to know that they do get this right and that it is based on Newton's Laws of Motion. For when DOTs choose to set yellow intervals shorter than this equation, they are in affect violating the immutable Laws of Motion, forcing cars to run red lights. This includes yellows intervals for left turn lanes, for the Laws of Motion apply to all places in the universe including left turn lanes.

One first must see how to derive the stopping distance equation in order to see how the ITE modifies it to arrive at their unsafe yellow interval:

The distance a car travels when the driver first sees the yellow light:

3. $S = x_p + d_s$

4. $S = t_p v_o + d_s$

Where

S = total distance (aka, critical distance) a car travels from when the driver observes a green light that just turned yellow to when the car comes to a stop.

$x_p = t_p v_o$ = distance car travels during the perception time

d_s = distance car travels while braking

t_p = the perception time = the number of seconds it takes the driver to observe and to initiate a response to a green light that just turned yellow

v_o = the speed of the car at the S

First solve d_s.

Using the equations of motion:

5. $v = dx/dt$
6. $v = v_o + at$
7. $S = x_p + \int v\, dt$
8. $S = x_p + \int (v_o + at)\, dt$
9. $S = t_p v_o + v_o t_a + a t_a^2/2$ When a is a constant
10. $d_s = v_o t_a + a t_a^2/2$
11. $t_a = (v_f - v_o)/a$ From 6, solve for t
12. $t_a = -v_o/a$ since $v_f = 0$ The final speed is 0 mph.
13. $d_s = v_o(-v_o/a) + a(-v_o/a)^2/2$ Substitute 12 into 10.
14. $d_s = -v_o^2/a + v_o^2/(2a)$
15. $d_s = -v_o^2/2a$

a = acceleration of the car (negative value is deceleration)
v_f = final velocity (0 = stopped)
v_o = initial velocity (the speed limit)
t_a = time it takes car to go from initial to final velocity

Plug d_s from equation 15 into equation 4.

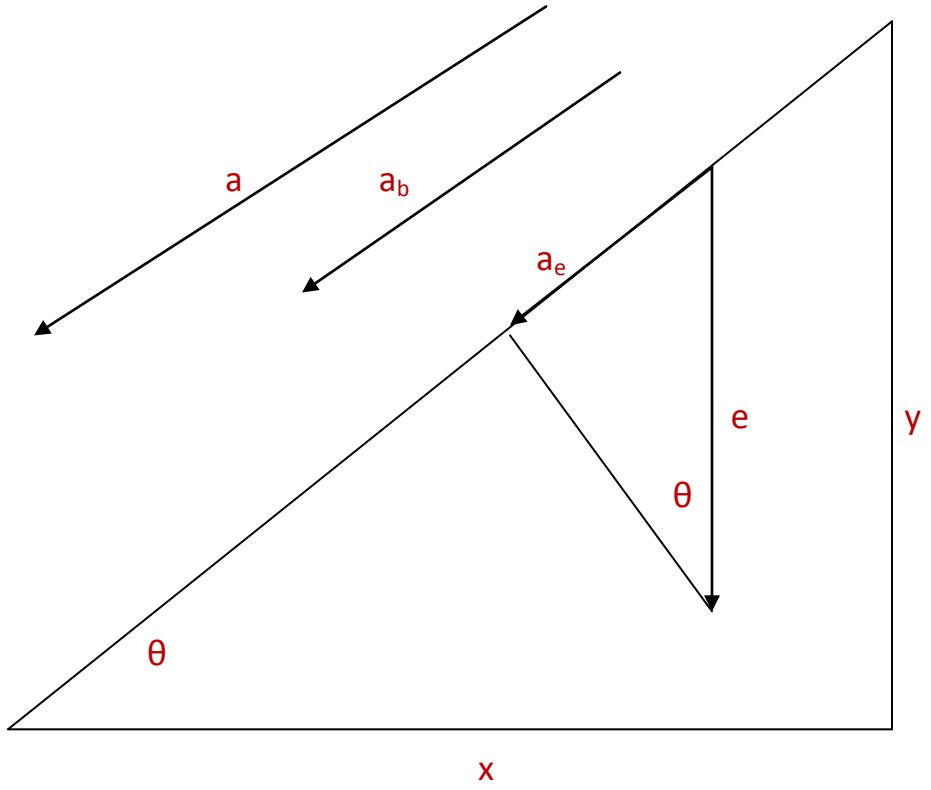
16. $S = t_p v_o - v_o^2/2a$

Let the acceleration now be a declaration. Set a = -a.

17. $S = v_o t_p + v_o^2/2a$

18. $S = v_o(t_p + v_o/2a)$

What contribution does the grade of the road add to the car's acceleration? What is the acceleration, a_e, to the car caused by Earth's gravitational acceleration?



19. $a = a_b + a_e$

Where

a_b = deceleration of the car due to the application of car's brakes
a_e = acceleration of the car due to force of gravity due to grade of road

ITE makes an assumption about a_b. Only a physicist would catch it. ITE assumes that on a level road, that the brakes of any vehicle can apply a force $\mathbf{F_b}$ resulting in a constant deceleration a_b. That means that no matter the mass of the vehicle, be it a Toyota Corolla or an 18-wheeler, the vehicle can always decelerate at a_b. That is why I can draw the above diagram using acceleration vectors instead of force vectors, the latter which a physicist would normally expect.

20. g = grade of road = rise over run = y / x

21. $g = y/x = \tan\theta$

22. $\theta = \tan^{-1}g$

23. $\sin\theta = a_e/e$

24. $a_e = e\sin\theta$

25. $a_e = e\sin(\tan^{-1}g)$

Using the small angle approximations, for small values of θ:

26. $\theta \approx \sin\theta$

27. $\theta \approx \tan\theta$

From equation 21 and equation 27:

28. $g \approx \theta$

From equation 26, substitute sinθ for g in equation 24:

29. $a_e = eg$

For small grades:

30. $S = v_o(t_p + v_o /2(a_b + a_e))$
31. $S = v_o (t_p + v_o /2(a_b + eg))$
32. $S = v_o (t_p + v_o /2(a_b + 32.2g))$

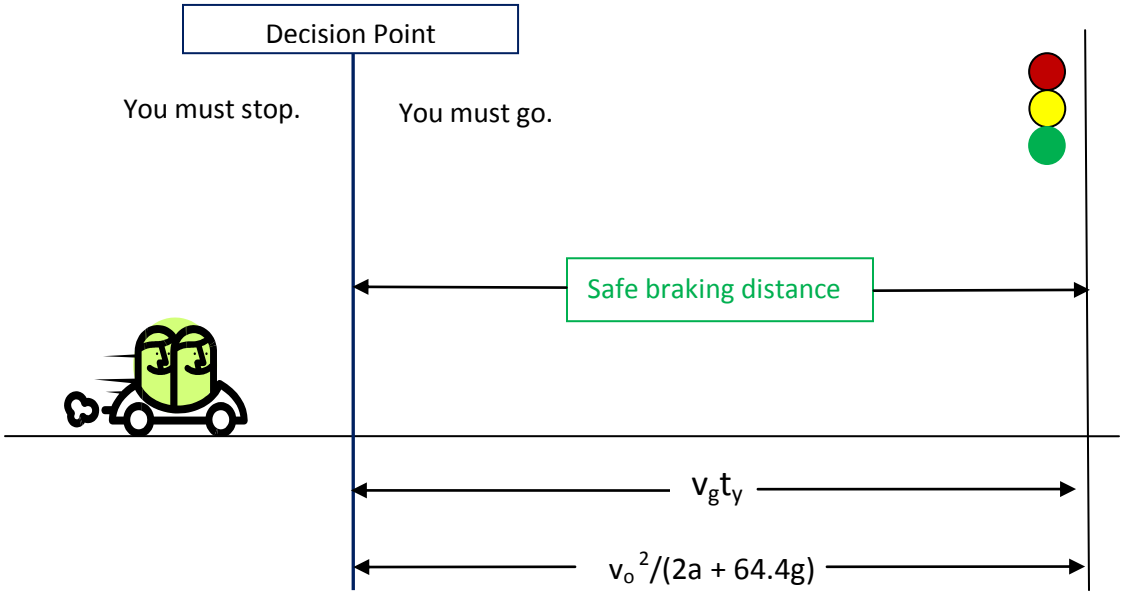
Because earth's gravitational acceleration is $e = 32.2\ ft/s^2$

$$\mathbf{S = v_o(t_p + v_o/2(a_b + 32.2g))}$$

is the *correct* formula for the distance from where the driver first sees the light turn yellow to where the driver comes to a stop at the intersection.

While the distance equation is right, the way the NCDOT and ITE computed how much time it takes for the driver to travel that distance is wrong. Instead of using a car that stops to traverse the stopping distance, they use a car that goes.

The False Premise at Work



The creators of the ITE's equation (eq. 35) use the safe braking distance point on the road before the intersection in order to determine how long the yellow light must be. But instead of using the amount of time it takes a car a stop to set the yellow time, ITE takes the amount of time it takes a car *that is not going to stop* to determine the yellow time.

The left side of equation 33 is the distance a car travels if he goes.

The right side of equation 33 is the distance a car travels if he stops.

33. $v_g t_y = v_o^2/(2a + 64.4g)$

v_g = the average velocity of a car that traverses the braking distance
t_y = yellow time for a car going v_g to traverse the braking distance

34. $t_y = [v_o^2/(2a + 64.4g)]/v_g$

Traffic engineers introduce an assumption at this point. They equate the velocity v_0 at the critical distance S of a stopping car, with the constant velocity v_g of a car proceeding to and entering the intersection. Engineers set:

35. $v_g = v_0$

It is possible to do this?

Yes it is.

Will this somehow create a problem for cars proceeding to the intersection at a constant velocity less than v_0?

No. Because $v_0/(2a + 64.4g)$ is $> v_g/(2a + 64.4g)$ when $v_0 > v_g$.

The yellow time set for a higher speed supports cars proceeding to the intersection at a slower speed. A slower moving car has a smaller critical distance. If a 5 second yellow is adequate for 50 mph approach speed, then a 5 second yellow is also adequate for a 45 mph car. The caveat is, is that the proceeding car must never decelerate. If a car starts at 50 mph on the approach, then decelerates to 45 mph, the formula opens up a type I dilemma zone. The formula can force the driver to run a red light.

Let's move on.

Apply equation 35.

Any physicist knows that the expression $v_o^{\,2}/(2a + 64.4g)$ defines a distance. Any physicist knows that the expression implies a time to stop which is $v_0/(a + 32.2g)$.

The yellow light time t_y is not that time. Let me explain.

In equation 33, equating the distance a travelling car which does not brake to the distance a stopping car travels, sets up the *time* it takes the car which does not brake to traverse the braking distance. It is the yellow time.

The math of equation 34 expresses the time it takes for a car that does not stop, to traverse the safe braking distance, at a constant speed v_o.

This seems like an oxymoron. It is confusing to everyone. It is confusing and paradoxical because it is merely the expression of a bad premise. What is being sacrificed here?

When you compute the yellow time this way, only at the distance $v_o^2/(2a + 64.4g)$ before the intersection, if the yellow turns yellow right at the point where the driver crosses that distance, then a driver can stop or go and not run a red light. If the driver is farther than that distance, then the driver must stop. If the driver is closer than that distance, then the driver must go.

At the distance $v_o^2/(2a + 64.4g)$ before the intersection, if the driver decides to go, t_y seconds later he will enter the intersection at the instant the light turns red.

At the distance $v_o^2/(2a + 64.4g)$ before the intersection, if the driver decides to stop, he is going to travel $v_o^2/(2a + 64.4g)$. When he comes to a stop, he will be stopped exactly at the intersection. The time it will take him to stop is $v_o/(a + 32.2g)$. This time is twice as much as the yellow time. Though it will take twice as long to stop, he will be able to stop before the intersection because all he is travelling is $v_o^2/(2a + 64.4g)$. It is just that half the time the driver is coming up to the intersection, the light will be red. Not only does the driver need all the yellow time to stop, but also he needs some red time.

Which brings us to ITE yellow light equation. Apply equation 35 to 34.

36. Yellow Interval = $t_p + v_o/(2a + 64.4g)$

Equation 36, though physically incorrect, is the equation one finds in ITE's Traffic Engineering Handbook[1] and the NCDOT Design Manual[1b]:

Yellow Interval = $t_p + v/(2a + 64.4g)$

Note that in ITE publications and subsequent DOT specifications, v_o turned into v. The concept of the *initial velocity* disappeared. ITE had miscopied Denos Gazis' equation[4] into the 1965 edition of the Traffic Engineering Handbook[3]. ITE has been propagating the error for 50 years. The explicit physical mandate that the velocity must be the velocity of the car at the start of the critical distance is now gone. Starting in 1994[5], perhaps before, traffic engineers decided they could stick in any velocity for *v* and miraculously get an adequate yellow interval. Traffic engineers started using average velocities anywhere along the approach, thus shorting the yellow interval, thus violating basic laws of physics, thus forcing even more people to run red lights.

Note that equation 36 is an equality. It is supposed to be an inequality. ITE miscopied Denos Gazis' formula again. Gazis said:

$$\text{Yellow Interval} \geq t_p + v/(2a + 64.4g)$$

By making the formula an inequality, ITE does not provide the driver a margin of error. The driver must know exactly when the mandate to stop changes into the mandate to go. If he guesses wrong by even $1/10^{th}$ of second, the formula will force him to run a red light.

Impractical and Dangerous

While the math exactly represents the false premise, one cannot apply the math without jeopardizing everyone's lives. The problems of this equation are:

A. It does not handle turning vehicles
B. No application of this equation handles turning vehicles. The equation in no way calculates any kind of time to decelerate.
C. It is impractical because you do not know the location of $v_o^2/(2a + 64.4g)$. It's guess work. The traffic engineer created the type II dilemma zone.
D. You have no option which guarantees your safety.
E. Safe braking is not always an option. You can get penalized for it.
F. If you think you have passed $v_o^2/(2a + 64.4g)$, but really you haven't and you decide to go, you will run a red light.
    a. You will run over a pedestrian.
    b. You will have a full-speed t-bone crash.
    c. You will get a red light camera ticket.
    d. You will get a ticket the old fashioned way—by a cop.
G. If you are inside $v_o^2/(2a + 64.4g)$, and you decide to stop, you no longer have the safe braking distance to stop.
    a. You will skid through the intersection **on a red**.
    b. Your head will go through the windshield.
    c. You will run over a pedestrian.
    d. You will have a low-speed T-bone crash.
    e. You will be rear-ended.
    f. You will get a red light camera ticket *for trying to stop*.
    g. You will get a ticket the old fashioned way—by a cop.

The Correct Yellow Light Interval Equation

Here's the correct derivation of the yellow light interval. This derivation is based on the correct premise that yellow light means brake, which means that yellow interval = stopping time.

37. $v_f = v_o + at$

38. $0 = v_o + at$ $v_f = 0$ because the final speed is a full stop

39. $v_o = -at$

40. Redefine a as a deceleration: $a = -a$.

41. $t = v_o/a$ = time it takes to come to a stop from the speed limit v_o

42. Yellow Light Interval = $t_p + v_o/(a_b + a_e)$

t_p = perception time
v = final velocity (0 = stopped)
v_o = initial velocity (the speed limit)
a_b = deceleration of car due to force of car's brakes
a_e = deceleration of car due to force of earth's gravity
e = acceleration of earth's gravity = 32.2 ft/s^2

From equation 42 and equation 25:

43. The correct yellow interval for all values of grade is:

Yellow Interval = $t_p + v_o/[a_b + e\sin(\tan^{-1}g)]$

From equations 42, 26 and 27,

44. The correct yellow interval for small values of grade

$$\text{Yellow Interval} = t_p + v_o/(a_b + eg)$$

Failures Even In the Correct Equation

The observant physicist will see a major limitation of all the yellow interval equations I have so far presented. All these equations apply only to dry roads. The equations are invalid when the road is slippery.

Rain or ice reduces the coefficient of friction of the road surface, which increases the safe braking distance, which increases the yellow light time. There is a formal mathematical expression for equation 43 which includes the coefficient of friction, but I do not present it here.

When it rains outside and you get a red light camera ticket, you can legally say, "Your yellow light duration doesn't account for when the road is slippery. It only accounts for dry conditions. You cannot judge my driving based on a red light camera that bases it decision on an equation that does not work in the rain. I am not God. I cannot stop the rain." See how far that gets. But that is exactly the case. As long as yellow light interval controller technology does not compensate for the realities of Nature, the judgment of an in-situ policeman remains necessary.

Just note that red light cameras enforce the law to the mathematical preciseness of the yellow light equation, whether or not the math represents reality. The equation also has physical demands which the traffic engineer must meet, one of which the engineer currently never meets. The engineer currently does not mark the road at the safe braking distance, which creates the dilemma zone.

Red light camera companies exploit the engineering failures. Cities shift the blame to the driver.

Conclusion

Equation 43 is what should appear in the *NCDOT Intelligent Transportation and Signal Systems Unit Design Manual* and in the *Institute of Transportation Engineers Traffic Engineering Handbook*. There is no need for equation 44, because there is no need for small angle approximations in the age of calculators.

The small angle approximation is satisfactory for grades between -10 and 10. But for grades outside those bounds, the small angle approximation gives less time than it needs to for inclines, and more than it needs to for declines. This is not a problem in North Carolina, but in San Francisco it would be.

As for the red light cameras, keep them. The red light cameras are the devices which caught the DOT with their pants down and the Town of Cary with their hands in the cookie jar. The cameras are independent monitoring devices. They are precision quality control instruments. What the Town of Cary did not expect and does not admit, is that the cameras caught the biggest offender of the law, and the biggest menace to public safety--the Institute of Transportation Engineers and the North Carolina Department of Transportation.

Once the towns set their yellow intervals to what physics demands, there won't be enough income from the program to sustain the program. There simply won't be enough people running red lights.

The Correct Yellow Intervals and Distances

Speed Limit (mph) v_0	Yellow Interval (s) $t_b = v_0/a$ $Y = 1.5 + t_b$	Braking Distance (ft) $v_0^2/2a = v_0t_b - \frac{1}{2}at_b^2$	Perception Distance (ft) $1.5v_0$	Stopping Distance (ft) $1.5v_0 + v_0^2/2a$
65	10.0	405.7	143	548.7
55	8.7	290.5	121	411.5
45	7.4	194.5	99	293.5
35	6.1	117.6	77	194.6
25	4.8	60.0	55	115.0
15	3.5	21.6	33	54.6

Where 1.5 s = perception time and a = the deceleration of 11.2 ft/s^2 as set by the standards of the NCDOT and AASHTO. These are the values for a level road. You will find the very same braking distances in AASHTO's *A Policy on Geometric Design of Highways and Streets*, 2004, p. 112. AASHTO perception and stopping distances are even more conservative than the ones listed above. AASHTO uses 2.5s for a perception time as opposed to NCDOT's 1.5s. AASHTO says 1.5s is good for an expected event, but on average, people need 1.0s more to react to an unexpected event.

As you see from this table, the math works. Note $t_b = v_0/a$ = braking time. The yellow intervals now accurately reflect the braking distances.

There are two on-line spreadsheets you can use to compute yellow light intervals. Both compute ITE and AASHTO "standards" as well as the correct values from Newton's Laws of Motion.

1. English System Yellow Light Intervals Spreadsheet
2. Metric System Yellow Light Intervals Spreadsheet

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About the Author

I own a software engineering company and a music company, Talus Software and Talus Music, in North Carolina. My engineering clients and employers have included NASA, The Lunar and Planetary Laboratory at the University of Arizona, ICAgen, Inc., General Electric, Engineering Technologies International, S & H Machine and Engineering, and believe it or not, the North Carolina Department of Transportation.

I graduated in 1983 from the University of Arizona with a bachelor of science in physics with a minor in astronomy.

It doesn't take a PhD in Mathematics to be an expert and affirm that 2 + 2 = 4. It doesn't take a PhD in Physics to be an expert and affirm that the ITE Yellow Interval Equation is bogus. All the material in this paper, the material in Denos Gazis' paper, and the material in all of ITE's and NCDOT papers are all topics completely covered in the first couple of weeks of an introductory physics class.

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References

[1] *Traffic Engineering Handbook*, 6th Edition, Publication TB-010B, Institute of Transportation Engineers, 2010, p. 412

The basic equation is found in every state, federal and international department of transportation's signals manual. The "2" in the denominator is the give-away.

Here are some examples:

[1a] *Traffic Signal Timing Manual*, Publication FHWA-HOP-08-024, Federal Highway Administration, 2008, p. 138

[1b] *Intelligent Traffic Signal Systems Unit Design Manual*, North Carolina Department of Transportation, 2009, Standard 5.2.2, Sheet 4 of 4

[1c] *Caltrans Traffic Manual*, California Depart of Transportation, Table 4D-102

[1d] *Signal Policy and Guidelines*, Oregon Department of Transportation, 2009, Appendix K

[2] *Traffic Engineering Handbook*, 5th Edition, Institute of Transportation Engineers, 1999, p. 481. This version of the formula has an algebra mistake. The 2nd and 3rd must be combined. The terms of velocity over acceleration are not additive; that is, they are not linear.

[3] *Traffic Engineering Handbook*, 3rd Edition, Institute of Traffic Engineers, 1965, p. 407.

[4] The Problem with the Amber Signal Light in Traffic Flow, Gazis, et. al, GM Research Labs, General Motors Corporation, Warren Michigan, 1959, p. 114. V_0 is the speed of the vehicle at the critical distance.

[5] *Determining Vehicle Signal Change and Clearance Intervals*, ITE Technical Council Task Force 4TF-1, Institute of Transportation Engineers, 1994, p. 4

[6] *The Dilemma with Dilemma Zones*, Tom Urbanik, University of Tennessee; Peter Koonce, Kittelson and Associates, p. 2.

[7] The Problem with the Amber Signal Light in Traffic Flow, Gazis, et. al, GM Research Labs, General Motors Corporation, Warren Michigan, 1959, p. 113, 116.

[8] Guidelines for the Preparation of Traffic Signal & Intelligent Transportation System Plans on Design-Build Projects, NCDOT, May 2009, p. 1

[9] The Problem with the Amber Signal Light in Traffic Flow, Gazis, et. al, GM Research Labs, General Motors Corporation, Warren Michigan, 1959, p. 128.