Yellow Change and All-Red Clearance Equations of Physics

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January 5, 2020

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The yellow change and all-red clearance equations in this document are those which conform to the laws of physics. The physics equations differ depending on the type of traffic movement. There is an equation applicable to unimpeded through-movements. There is a different equation applicable to unimpeded turning movements. And there is a different equation applicable to impeded movements. This third equation, the one golden equation, can be applied to any yellow change interval to encompass all drivers in all situations and traffic movements. The equations change when the driver ascends a hill, descends a hill or travels on a level road. The equations differ whether the jurisdiction enforces a permissive or restrictive yellow law.

With equal importance, this document shows the mathematical technique of error propagation. Error propagation calculates tolerances. All fields of engineering use tolerances to express the uncertainty in a dimension. Traffic engineering should follow the practice of the other fields. Because perception-reaction time, approach speed and deceleration are not precise, the yellow change interval, a dimension of time, is not precise. It has a tolerance—a range of acceptable values. Computing tolerances is necessary for law enforcement, especially automated enforcement, so as not to punish drivers who are innocently entering the intersection on a red light within the time of the tolerance.

Definitions

Variable	Description
Y_{min}	Minimum duration (seconds) of the yellow light for a jurisdiction with a permissive yellow law. Round calculated value up to nearest 0.1 second.
Y_{min-r}	Minimum duration (seconds) of the yellow light for a jurisdiction with a restrictive yellow law. Round calculated value up to nearest 0.1 second.
c	Critical distance (ft). Also known as the safe and comfortable stopping distance. It is the distance the vehicle travels during time t_p plus the distance the vehicle safely and comfortably decelerates to a stop.
t_p	Perception + reaction + air-brake lag + line-of-sight obstruction times (seconds). Depends on complexity of intersection. Simplest intersection without obstructions blocking view of traffic signal is typically 1.75 seconds. 0.75 seconds is typical air-brake lag time for commercial vehicles.

v_c	Approach speed (ft/s). It is the speed of freely-flowing vehicles measured at the critical distance upstream from the intersection. Vehicles have not begun to decelerate toward the intersection. $v_c \geq$ posted speed limit, preferably the 85th percentile speed.
v_e	The velocity (ft/s) the vehicle enters the intersection
v_{avg}	Average velocity (ft/s) of vehicle from the critical distance upstream from the intersection to the intersection entrance
v_x	Average velocity (ft/s) of vehicle as it travels inside the intersection
a	Safe and comfortable deceleration (ft/s^2). Typically 10 ft/s^2
Γ	Deceleration contribution due to gravity on account of grade
H	Deceleration contribution due to gravity on account of grade on a vehicle ascending a hill
g	Earth's acceleration due to gravity on an object in free fall = 32.2 ft/s^2
G	Grade of road (rise over run, negative values are downhill). Grade is percent / 100.
R	All-red clearance interval (seconds). Round calculated value up to nearest 0.1 second.
L	Length of vehicle (ft)
P	Length of path from intersection entry point to intersection exit point.
D	Red light camera delay; aka, grace period; aka, tolerance, uncertainty, (seconds). Law enforcement should not ticket a driver unless driver runs the red light exceeding this amount of time.
$\Delta \boldsymbol{t_p}$	Uncertainty which is half the known range (seconds) for t_p. Typically 1.5 seconds. AASHTO max 3.5 seconds. Gazis low = 0.5 seconds.
$\Delta \boldsymbol{a}$	Uncertainty which is half the known range for safe and comfortable deceleration (ft/s^2). Approximately 2.0 ft/s^2. (Commercial trucks, snow, ice: min = 8.0 ft/s^2. Transportation Research Board, Wisconsin: max = 12.0 ft/s^2.)
$\Delta \boldsymbol{v_i}$	Uncertainty which is half the known range for the intersection entry speed (ft/s) for left and right turning vehicles. The range in intersection entry speed on a 45 mph road ranges from 10 mph to 35 mph. Half the range is 18.2 ft/s.

Yellow Change Interval

To properly compute a yellow change interval, the engineer has to 1) define the critical distance, 2) describe the motion a proceeding vehicle takes through the critical distance, then 3) calculate the time it takes for that proceeding vehicle to perform that motion. In that order.

Critical Distance

This equation represents the distance "**c**" a driver needs to stop with safe and comfortable deceleration. **c** is called the critical distance. The point on the road at distance "**c**" upstream from the intersection is called the critical point, or the point of no return. **Γ,** contribution by gravity due to grade of road, becomes 0 when ascending a hill.[1]

$$\boldsymbol{c = t_p v_c + \frac{v_c^2}{2[a+\Gamma]}} \quad \boldsymbol{\Gamma = \begin{cases} g\sin(\tan^{-1} G), & G \le -0.1 \\ gG, & -0.1 < G < 0 \\ 0, & G \ge 0 \end{cases}} \qquad \text{(Eq. 1)}$$

gG, an expression found in yellow change interval formulas, is the small angle approximation of $\boldsymbol{g\sin(\tan^{-1} G)}$.

Yellow Change Intervals for Jurisdictions with Permissive Yellow Law

The minimum yellow change interval is the time it takes a driver to traverse the critical distance. The minimum time will enable the driver to enter the intersection legally. If the driver is at c when the light turns yellow, the light will turn red the instant the driver enters the intersection.

Formula for Unimpeded Through-Movements on a Level Road, **(Gazis, ITE)**

For unimpeded through-movements, after the driver crosses the point of no return, the driver must traverse the entire critical distance at the maximum allowable speed v_c or faster (beat the light), in order to enter the intersection legally.

$$Y_{min} = \frac{c}{v_c} = t_p + \frac{v_c}{2a} \quad \text{(Eq. 2)}$$

Formula for Unimpeded Through-Movements Descending a Hill, **(ITE)**

$$Y_{min} = \frac{c}{v_c} = t_p + \frac{v_c}{2[a + \Gamma]} \quad \text{(Eq. 3)}$$

Formula for Unimpeded Through-Movements Ascending a Hill

When ascending a hill, gravity works against the driver increasing his time to traverse the critical distance.[2]

$$Y_{min} = \frac{-v_c + \sqrt{{v_c}^2 - 2Hc}}{-H} \qquad H = \begin{cases} g\sin(\tan^{-1} G), & G > 0 \\ gG, & 0 < G < 0.1 \end{cases} \quad \text{(Eq. 4)}$$

Formula for Turning Traffic (Left, Right, U) (Liu, ASCE)[3]

This formula divides the critical distance by the average velocity. The formula assumes constant deceleration from distance c upstream from the intersection to the intersection. There are other ways turning drivers approach the intersection but Liu's equation covers them all.[4]

$$Y_{min} = \frac{c}{(v_c + v_e)/2} \quad \text{(Eq. 5)}$$

Formula for Impeded Traffic

Use the following formula when there are reasons vehicles slow down (other than for stopping) within the critical distance on route to the intersection. Business entrances, near-by intersections, lane-changing for an upcoming fork, higher traffic density within the intersection, speed limit reductions near the intersection, railroad tracks near or in the intersection and unexpected hazards all cause drivers to slow down.

$$Y_{min} = \frac{c}{v_{avg}} \quad \text{(Eq. 6)}$$

General Formula That Works For All Traffic (Newton, Ceccarelli, Shovlin)[5]

The following formula is the deterministic equation which works for all traffic movements. This formula is the stopping time.

$$Y_{min} = t_p + \frac{v_c}{a + \Gamma} \quad \text{(Eq. 7)}$$

The formula is the general equation of motion for any non-relativistic object traveling in a straight line with constant acceleration in a reference frame in uniform motion. You will find the formula, without the t_p, in the first chapter of an introductory physics book. This formula gives unimpeded through-movement traffic a few more seconds than needed, but gives turning (a U-turn) and impeded movements the minimum amount of time.

If adopted, this equation comes with a new directive to drivers. "At the moment the light turns yellow, an action of decelerating as if coming to a comfortable stop always will be possible without running a red light. The most unusual situation that can occur is that you arrive at the stop bar stopped while the light is still yellow. When that is the case, wait a second for the light to turn red."[6]

Other Formulas

Many yellow change and all-red clearance equations have been proposed other than the ones in this document. These other equations will not be considered in this paper because they are not physical solutions.[7]

All-Red Clearance Interval for Jurisdictions with Permissive Yellow Law

$$R = \frac{L + P}{v_x} \quad \text{(Eq. 8)}$$

Yellow Change Interval for Jurisdictions with Restrictive Yellow Law (ITE)

A restrictive yellow law means that a vehicle cannot be in the intersection on a red light. The vehicle must clear the intersection while the signal is still yellow or green.

$$\boldsymbol{Y_{min-r} = Y_{min} + R} \qquad \text{(Eq. 9)}$$

All-Red Clearance Interval for Jurisdictions with Restrictive Yellow Law (ITE)

Theoretically you can set R to 0 but most jurisdictions set R = 1 second for safety.

$$\boldsymbol{R = 0} \qquad \text{(Eq. 10)}$$

Tolerance of the Yellow Change Interval

The traffic engineering community should adopt the standard engineering practice of tolerances. A tolerance of a dimension is the known “error in the dimension” or the “uncertainty in the dimension.” Tolerance, error and uncertainty mean the same thing. A yellow change interval, a dimension of time, has an uncertainty because its constituent variables such as perception-reaction time and deceleration have uncertainties. One computes the tolerance of the calculated yellow change interval using the standard mathematical technique of error propagation. The engineer *propagates* the constituent uncertainties to the calculated value.

The purpose of computing the tolerance is to instruct law enforcement not to punish drivers if they run the red light within the tolerance.

Perception-reaction time and deceleration are not constants in the physics meaning of the word “constant.” There is not a unique value in the universe for perception-reaction time. There is not a unique value in the universe for deceleration. Instead perception-reaction time and deceleration each represent a *range* of *equally valid* values. For example, a 25 year-old driver has one perception-reaction time. A 70 year-old driver has a different one. Both are equally valid because both drivers are allowed on the road. Further, perception-reaction time may vary for the same person depending on conditions present at a particular moment in time. The ranges represented by the “constants” composing the yellow change interval equation contribute to the range of the yellow change interval, giving the interval an equally-valid range of values. Law enforcement should not treat the yellow change interval as if it is an universally unique constant and thereby enforcing it to precision.

Stochastic techniques are the application of statistical functions to determine means, percentiles and standard deviations. Stochastic techniques apply only to random events. For many decades, traffic engineers have set perception-reaction times and decelerations to means or percentiles. But one cannot use stochastic techniques to determine values such as perception-reaction times and decelerations. Determining the mean, 85th percentile or standard deviation and then adopting them without qualifying that there is a tolerance or range of acceptable values infers the existence of one and only one valid value. Perception-reaction times and decelerations, however, are neither random nor do they have one valid value.[8]

The preferred way for an engineer to look at perception-reaction time, deceleration and vehicle entry speed is to use either the maximum or the minimum value (whichever value contributes to the longest yellow) for each range. Only in this way can the engineer safeguard (see *Scope* in Notes section) the public's life, health and property. In the absence of the preferred way, the engineering community should at least adopt the standard engineering practice of listing tolerances to minimize unfair legal prosecution.

The tolerance equations (below), require a value and the uncertainty for each "constant." The engineer uses the midpoint of the range for the "constant" and half its range for the uncertainty. The engineer can then compute the yellow change interval using the midpoints, and the yellow interval's uncertainty using the appropriate tolerance equation. The engineer writes the computed tolerance value, D, in his Timing Chart on the signal plan of record and instructs law enforcement not to punish drivers if they run the red light within the tolerance.

If the jurisdiction has red light cameras, then law enforcement must set the red light camera delay (aka. grace period) to D; otherwise, the minority of drivers who are not conforming to the chosen "constants" but nonetheless are acting reasonably will be subject to unfair penalties.

General Form of Computing the Red Light Camera Delay

D is the tolerance of the yellow change interval where the function ***Y*** is a yellow change interval equation. Equation 11 shows the mathematical technique of error propagation.

$$\boldsymbol{D} = \Delta \boldsymbol{Y}_{\boldsymbol{min}} = \left|\frac{\partial Y}{\partial t_p}\Delta \boldsymbol{t}_{\boldsymbol{p}}\right| + \left|\frac{\partial Y}{\partial a}\Delta \boldsymbol{a}\right| \ldots \qquad \text{(Eq. 11)}$$

The math of error propagation answers the question, “Given an equation whose constituent variables each have an uncertainty, how do these individual uncertainties change the uncertainty of the computed value?” In calculus the word “change” is key. “Change” implies to take the derivative. A change with respect to a specific variable, is the derivative of the equation with respect to that variable. Therefore the contribution of that variable to the overall tolerance, equals the derivative of the equation with respect to that variable, multiplied by the uncertainty of the variable. One sums the contributions of each constituent variable to get the tolerance of the computed value. (One does not use quadrature because perception-reaction time and deceleration are not unique values whose measurements are Gaussian distributions.)

Delay Formula for Unimpeded Through-Movement on a Level Road under a Permissive Yellow Law

Assume the error in v_c and G, P are negligible. Ignore error in L. Eq. 12 shows the derivative of Gazis’ equation with respect to perception-reaction time + the derivative of Gazis’ equation with respect to deceleration, each derivative multiplied by its respective uncertainty and then added:

$$\boldsymbol{D = \Delta t_p + \frac{v_c}{2a^2}\Delta a} \qquad \text{(Eq. 12)}$$

The value of D is about 2.3 seconds on a 45 mph level road for unimpeded through-movements.[9]

Delay Formula for Turning Movements under a Permissive Yellow Law

Assume the error in v_c and G, L and P are negligible. Ignore error in L. This equation reveals the derivatives of Liu’s turning equation.

$$\boldsymbol{D = \left|\frac{2v_c}{v_c + v_e}\Delta t_p\right| + \left|\frac{{v_c}^2}{a^2(v_c + v_e)}\Delta a\right| + \left|\left(\frac{2v_c\left(t_p + \frac{v_c}{2a}\right)}{(v_c + v_e)^2}\right)\Delta v_i\right|} \qquad \text{(Eq. 13)}$$

The value of D is about 3.4 seconds on a 45 mph level road for unimpeded turning movements.[9]

Safety, Legal Motion and Negligible Contributions to Tolerance

- The purpose of adopting the practice of writing tolerances in the signal plan of record, is to prevent law enforcement from punishing innocent drivers. This has significant effect. Red light cameras have already punished tens of millions of drivers who have entered into the intersection on red within the tolerance of the yellow change interval.

- Tolerances do not enhance safety; lengthening the yellow change interval enhances safety. The tolerance value itself does not change what the driver sees when approaching a traffic signal. Lengthening the yellow change interval does.

  Regardless of what equation the engineer uses, implementing the maximum-minimum value for each range is the only way to dispense with tolerances.

- Equations 12 and 13 say that the uncertainty in v_c is negligible. What is meant is that v_c is the only variable over which traffic engineers have some control. In reality there is an uncertainty in v_c and it does not hurt to propagate that uncertainty. Also consider the uncertainty in a speedometer. According to federal regulation 49 CFR III Sec 393.82, a speedometer must be accurate to within +/- 5 mph at 50 mph.

- There is an uncertainty in G, the grade measurement, and you can propagate that uncertainty. G's uncertainty contributes to the overall uncertainty significantly less than perception-reaction time and deceleration.

- For restrictive law States, it is important to propagate the uncertainty in v_x.

History

Many of the above equations may be new to the traffic engineering community. Most engineers have used only the ITE equation. Current practice, even as written in the recent study NCHRP 731, is to mistakenly apply the ITE equation to every traffic movement. The last surviving inventor of the ITE equation, Alexei Maradudin, in letters to CalTrans and to ITE, and in an ABC TV interview, unambiguously clarifies that the ITE equation should only be used for unimpeded through movements. The ITE equation is based on his original theories and research.

From the ABC Interview:

> *We did not, in our analysis consider turns; either left hand turns or right hand turns. It was really straight through the intersection dynamics that we considered.*

From the letter to CalTrans:

> *The formula can only be applied to vehicles which start at the maximum allowable speed measured at the critical stopping distance and which proceed at a constant speed into the intersection. ...The formula does not work for any other circumstance.*
>
> *Applying the formula to circumstances where a driver must decelerate within the critical distance into the intersection results in a minimum amber time which is shorter than what is necessary to eliminate the dilemma zone. Below is a partial list of common situations where the formula does not provide a long enough minimum amber time:*
>
> *1. Traffic turning left where the speed limit is greater than the intersection entry velocity.*
>
> *2. Traffic turning right where the speed limit is greater than the intersection entry velocity.*
>
> . . .
>
> *5. Traffic preceding straight that slows down for vehicles entering and existing the roadway to and from business entrances and side-streets near the intersection.*
>
> . . .

From the letter to ITE referring to ITE's draft for the recommended signal timing practices:

> *This passage appears to suggest that the methods we used in our 1959 study can be used to obtain results for closely spaced signals at a divided highway or other variations such as turning movements. This would not be a correct interpretation of our work. Our methods are applicable only to through movements where drivers are able to maintain their speed, not on roadways with closely spaced signals or for turning movements.*

In 1996, Dr. Chiu Liu, M.ASCE, a physicist and civil engineer at CalTrans, wrote with Gazis and Herman, "A Review of the Yellow Interval Interval Dilemma" as a follow up to Gazis' and Herman's work. In the 1996 paper, Liu, Gazis and Herman wrote,

> *In this paper, we attempt to clarify the problem of the yellow interval dilemma. Since quotations and misinterpretations of eqn (9) of the GHM paper have appeared in my different versions of the Institute of Transportation Engineers (ITE) handbooks and various research journals . . . .*

In 2002, in collaboration with Gazis and others, Liu addressed the problem of left turns in ASCE's peer-reviewed Journal of Transportation Engineering.

> *The setting of the yellow change and red clearance interval for straight movements has been reviewed, and explicit formulas . . . are discussed in detail. However, the setting of these signal intervals for turning movements has not been understood in both theory and practice (ITE 1985).*

Notes

SSD Grade Compensation vs Signalized Intersection Grade Compensation

1 In 1982, Kell and Fullerton incorrectly took the term gG from the stopping sight distance (SSD) formula. The problem is that gG in the SSD formula applies only to stopping distances in emergency conditions.[10] Only when the driver presses his brakes as hard as he can does gravity directly subtract from (downhill) or add to (uphill) the vehicle's deceleration.

Under conditions of a normal approach (not emergency conditions) to a signalized intersection, grade's contribution to deceleration differs from that in the SSD equation. Consider a stopping driver. When a driver descends a hill, should he brake harder than he does for a level road? Should the driver press his brake farther towards the floorboard? The driver does not want to press the brake harder than he does for a level road. While the driver's vehicle may be capable of exerting extra braking force, it is not an action the driver puts his vehicle through routinely. The driver does not expect that he should have to do it. The driver anticipates that drivers behind him do not expect it either. Not only does a driver expect consistency in comfortable deceleration but also consistency in the demands put on his vehicle. Therefore for a downhill grade, $\boldsymbol{\Gamma}$ applies and $\boldsymbol{\Gamma} < 0$. Kell and Fullerton accidentally added the correct algebraic expression to the yellow change interval equation.

However Kell and Fullerton are incorrect when a driver goes up a hill. Consider the stopping driver. The driver does not press the brake as hard as on a level road. That would decelerate him uncomfortably. The driver unconsciously works to attain a consistent deceleration going up a hill as he does on a level grade. Therefore ascending a hill, $\boldsymbol{\Gamma} = 0$--the uphill case of the emergency SSD formula does not apply.

Quadratic Equation for Time to Ascend a Hill

2 The question is, "How long does it take to traverse the critical distance ascending the hill?" Though gravity does not work to change the uphill human-desirable stopping deceleration, it does have the effect of increasing the time to traverse the critical distance. The solution takes the form of the quadratic equation (eq 4). Y_{min} is the solution for t in the equation $-½Ht^2 + v_ct = c$. The equation is a polynomial of degree 2 thus solvable using the quadratic equation. "t" the amount of time it takes for the vehicle to traverse the critical distance. The left side of the equation is from

Newton's second law. It is the distance traveled by a vehicle with initial approach velocity v_c under constant deceleration H. H is the deceleration which the grade contributes against the vehicle ascending the hill. The right side of the equation is the critical distance.

It is crucial to acknowledge the bifold purpose of the yellow change interval. The yellow change interval equally concerns both 1) the distance to stop and 2) the time to traverse the critical distance within the time the signal light remains yellow.

When ascending a hill to an intersection, drivers who are too close to stop keep going but they unconsciously slow down while approaching. Drivers are too focused on events at the intersection to think about the need to maintain a constant velocity. (Drivers are not even aware that the ITE equation requires them to maintain a constant velocity.) Gravity slows them down--an act which the ITE equation does not handle. Drivers in this situation take longer to traverse the critical distance than on a level road. Therefore using the ITE equation makes the driver's situation worse. The ITE equation makes the yellow light shorter than that for a level road but drivers going uphill actually need more yellow time, not less.

Meaning of Liu's Turning Equation

3 **A.** Liu's equation assumes that the driver at critical point "c" upstream from the intersection starts to slow down into the intersection.

B. The equation assumes the driver slows down into the intersection at a constant deceleration. Because the driver enters the intersection at speed greater than zero and starts his decelerating before his perception-reaction period is over, his deceleration is less (often much less) than the comfortable stopping rate. The rate can easily be 4 to 6 ft/s^2.

C. The value the engineer uses for perception-reaction time in Liu's equation is the same as for Gazis' through-movement equation. All the preconditions are the same. In both cases the vehicle approaches the intersection at constant velocity v_c. In both cases the vehicle crosses the critical point at constant velocity v_c. In both cases the driver has his foot on the gas. In both cases the driver is faced with the same stop-go decision.

The perception-reaction time is not the "perception-reaction time *while* decelerating." The driver is not yet decelerating. His foot is not yet on the brake.

For both through and turning movements, c is the point of no return. Once closer to the intersection than distance c, though a driver may still have the braking distance to stop, he no longer has enough perception-reaction time to make the decision to stop. It takes the driver the entire perception-reaction distance and the braking distance to be able to stop. Once beyond the critical point, the driver must proceed into the intersection.

Alternate Turning Equations

4 **A.** Consider the turning driver with no queue in front on him. The most aggressive turning driver goes as fast as legally possible for as long as he can and then at the last possible moment, decelerates into the intersection at the quickest comfortable deceleration. t_p is the though-movement perception-reaction time because the driver has crossed over the critical point at v_c and his foot is not yet on the brake. This equation is:

$$\boldsymbol{Y_{min}} = \mathbf{t_p} + \left[\frac{\mathbf{v_e^2}}{2\boldsymbol{v_c}(\mathbf{a} + \boldsymbol{\Gamma})}\right] + \frac{\boldsymbol{v_c} - \mathbf{v_e}}{\mathbf{a} + \boldsymbol{\Gamma}} \qquad \text{(Eq. 14)}$$

While this equation is a physical solution, it is not needed. Liu's formula covers this possibility.

Liu's equation is the *boundary condition*. It represents the *slowest* reasonable traversal of the critical distance. Equation 14 expresses the other boundary condition. Equation 14 represents the *fastest* reasonable traversal of the critical distance. The slowest traversal provides enough time for the fastest traversal so the fastest is moot.

Eq. 14 has appeared on the ITE forum and some traffic engineers have grabbed a hold of it. Eq. 14, like the ITE equation, is a *partial* physical solution.

B. Consider a different case of turning. Many traffic engineers consider the case when the driver is already decelerating when the light turns. Engineers assume that such a driver has a smaller perception-reaction time. Engineers assume that the decelerating driver takes less perception time to decide to stop than a through-movement driver, and/or a lesser reaction time because the driver's foot is already on the brake. One can argue the validity of the assumptions but such arguments are moot. Just like equation 14, Liu's equation is the boundary condition which covers the case.

Because many engineers are interested in the case of the already-decelerating driver, let us set up the mathematics, but not solve it:

i. To solve any yellow light equation, one sets up an equality between the distance it takes a driver to stop and the distance it takes a driver to proceed. Then one solves for t—the time to proceed. The time to proceed is the yellow change interval.
ii. Consider the critical distance. The critical distance is the perception-reaction time distance plus the braking distance. The critical distance will be a function of proceeding deceleration, stopping deceleration and initial velocity.
iii. Consider the proceeding distance. The distance is how far the driver proceeds when he intends to enter the intersection. This equation will be a function of t. Eventually you will solve for t.
iv. The critical distance would be the sum of a smaller perception-reaction distance and a smaller braking distance.
v. The perception-reaction distance is smaller because the driver is decelerating during the perception-reaction time.
vi. The braking distance is smaller because the initial velocity at the start of braking is smaller due to deceleration during the perception-reaction time.
vii. Given that driver starts at speed v_c, decelerates at rate β and enters the intersection at v_e,
    a. formulate the new critical distance equation
    b. formulate the traversal distance equation
viii. Set the critical distance equation equal to traversal distance equation.
ix. Solve for the deceleration β. Once you have the expression for β, you can plug it back into the critical distance equation and a get an expression for the critical distance based on v_c, v_e and a.
x. The critical distance = traversal distance formula is a cubic equation and β is one of the roots of that cubic equation.
xi. Solve for β.
xii. Plug β into the critical distance expression.
xiii. Compute the yellow change interval. Divide the critical distance by the average velocity $(v_c + v_e) / 2$ through the critical distance.

5 Ceccarelli, Shovlin. *Derivation of the Yellow Change Interval Formula*. This paper shows the derivation from Newton's second law, Gazis' unimpeded through-movement equation on a level road, ITE's extension of grade, Liu's turning equation, and the general equation; that is the equation for impeded movement, the latter to which all objects in the universe conform.

6 Equation 7 removes the mutually exclusive choice of stop or go therefore eliminating all dilemma zones. Both type 1 (no solvable stop-go solution) and type 2 (indecision) dilemma zones disappear because there is no critical point. The traditional critical point, where stop turns into the mandate to go, disappears for there is no single point on the road where the driver, upon seeing the light turn

yellow, must make one choice or the other. Indecision disappears because stopping is always a valid decision.

As before, going will be a valid decision when the driver decides he does not have the distance to stop. The difference now is that the driver now always has the time to decelerate into the intersection. It takes less time to traverse the fixed stopping distance crossing the stop bar at a final speed greater than 0 than it does arriving at the stop bar stopped. The yellow time is now the time to stop.

Using equation 7, when a driver approaches a signal and the light turns yellow and he decides he has just enough distance to stop, the light turns red at the same time he reaches the stop bar. For the current formula (equation 2), the light turns red several seconds before the driver reaches the stop bar.

Traffic engineers worry over creating the opposite of dilemma zones. Option zones. The fear is while drivers approach, some drivers will want to go while others want to proceed. Having both options available may create unpleasant interactions. Therefore equation 7 comes with an education directive. The public must learn how to react.

Engineers fear that drivers may disrespect a longer yellow light. Whatever that exactly means, it has been a popular hypothesis for over 70 years. It was debunked in 1961. "The results lend no support to a popular hypothesis, i.e., that drivers tend to "take advantage" of a long amber phase by treating it as an extension of the green" is in the abstract of Olsen and Rothery's paper, *Driver Response to the Amber Phase of Traffic Signals*. General Motors Laboratories, 1961)

From the physics point-of-view, the solution is clear. Equation 7 is the only solution. It is the only one capable of accommodating allowable traffic movements which always appear in all traffic lanes.

Physical vs Stochastic Solutions, Analytic Solutions and Misapplications of Physics

7 The practice of engineering is the application of the physical sciences; those being, chemistry, earth science and physics. This definition appears in most States' statutes like California Professional Engineers Act 6701, Florida Statutes Title XXXII, Chapter 471.005(7), North Carolina General Statute 89C-3(6), Texas Statute Title 6, 1001.003(b) and 137.51, as well as in Merriam Webster's Dictionary and the ITE Constitution.

Physical solutions are the outcome of the application of the physical sciences. Stochastic, analytic and misapplications of physics are not. The engineer must avoid stochastic solutions, analytic solutions and misapplications of physics for such practices are not engineering practices.

Below is a description, mainly by example, of each kind of solution.

Physical Solutions

A physical solution is one that manifests itself in the real world. A physical solution is one that conforms to and is explainable by the laws of the physical sciences. When an engineer's solution conforms to the scientific method, the engineer produces a physical solution.

Example 1: A physical solution is the ITE change interval equation. The ITE equation is the true-to-physics representation of an unimpeded through-movement vehicle. The equation works all the time under the preconditions that the driver knows the exact location of the critical point, approaches on dry pavement towards a signalized intersection, conforms to whatever constants the engineer plugs in for perception-reaction time, maximum allowable speed (v) and deceleration, and if he decides to go, he goes at the maximum allowable speed all the way into the intersection. When all the preconditions are met, the ITE equation is a proper physical solution.

Scope of the Physical Solution Must Include Everyone

A *professional* engineer, as in the legal credential P.E., is an engineer with more scope than the electrical engineer who designs circuit boards for PlayStation. While all engineers apply the physical sciences to their creative work, a P.E.'s work must additionally "safeguard the life, health and property of the public." This particular phrase is found in the same statutes as the definition of engineering practice. (See above, the first paragraph of note 7.)

The word "public" defines the scope of the physical solution. The PlayStation engineer does not have to design his work so that everyone has to be able to use it. But the P.E. does. That is the difference. The State holds the P.E. to a higher standard. Because every person is a bona fide member of the public, whatever physical solution the P.E. provides, it must accommodate all allowed vehicles on the road, accommodate all reasonably perceptive people, and accommodate all allowable traffic movements.

In the language of physics, P.E.s must accommodate boundary conditions.

Example 1: Violation of Scope. A structural engineer is designing a building in San Francisco. From geologic data, he computes the average strength of an earthquake to be 3.5 on the Richter scale. Though the engineer knows that hundreds of earthquakes with strengths greater than 4.0 occur every year, he designs his building to withstand only the average earthquake.

Example 2. Violation of scope. Many DOTs use 11.2 ft/s^2 as the deceleration for the yellow change interval. But 11.2 ft/s^2 according to AASHTO[11], is the 90th percentile deceleration under *emergency* conditions. In concept this value excludes all drivers stopping normally. By definition, the engineers working for the DOTs are considering the scope of the "public" as only those drivers who stop under emergency conditions and not considering those drivers that stop normally.

11.2 ft/s^2 emergency stopping is about the equivalent of the 65th percentile of normal decelerating. That means an engineer can expect only 35 out of every 100 drivers to stop this quickly for a traffic signal.

When set to 10 ft/s^2, the engineer can expect only 50 out of every 100 people to stop this quickly.

The engineer knows that 10 ft/s^2 applies only to passenger vehicles. He knows that 8.0 ft/s^2 is the comfortable deceleration value for commercial vehicles.[8]

Example 2. Violation of scope. Dr. Lei Yu wrote TxDOT Report 0-4273-2. It is an interesting report in the fact that Yu favors traffic efficiency at the sacrifice of the legal motion of traffic, *knowingly*. Yu knows that left-turning vehicles need more yellow time than through movements in order to enter an intersection legally. Yu knows this because Liu and Yu wrote the earlier paper with the turning equation. Yu was coauthor. The conclusion of the earlier turning paper is that turning yellows must be longer than through yellows.

In the newer TxDOT report, Yu does use the turning equation. But Yu manages to shorten the turn yellows to values less than through-movements. How did Yu accomplish this? In the TxDOT report, Yu used stochastic methods and a poll of traffic professionals. Both are not engineering practices, but he used both to circumvent his previous conclusion. The poll ranked the legal motion of traffic *seventh* out a list of 10 concerns. To make his numbers fit the poll, Yu used stochastic methods on non-random events and plugged the results into the turning equation. Yu did not measure vehicle speeds at the critical distance and did not use the maximum allowable speeds. And Yu set perception-reaction times to 0 and averaged queued vehicles with unimpeded vehicles.

Yu ignored the boundary conditions. Yu's earlier paper forbade this kind of treatment. Yu ignored the safeguarding of the public from unfair legal prosecution and did not inform the Texas police that his yellow timing protocol will ensure that some portion of the motoring public will inadvertently run red lights.

Misapplications of Physics

Example 1. Since 1965, ITE has taken Gazis' specialized equation and has been applying it to *all* types of traffic movements. However we know from the physics of Gazis' equation, Gazis' paper and from unambiguous comments from Alexei Maradudin (coauthor of Gazis' paper), that Gazis' equation applies only to unimpeded through-movements confined to specific preconditions. NCHRP 731 continues to misapply Gazis' equation to left-turns. The problem is that unimpeded left-turning vehicles do not move at constant speed as required by Gazis' equation. Unimpeded turning vehicles decelerate into the intersection after passing the critical point. (Only where the maximum allowable speed is around 20 mph would a left-turning vehicle's speed be constant through the critical distance thereby allowing Gazis' equation to be used.) The mathematics to describe a decelerating vehicle is different than Gazis' equation. Liu's mathematics properly describe the boundary conditions of a turning vehicle. Turning vehicles require several seconds more than through-movement vehicles.

Example 2. Many traffic engineers set "v" in the yellow change interval equation to the speed measured at the stop bar. Physics says the "v" in the yellow term of the polynomial is the free-flow speed measured at the critical distance upstream from the intersection. It is not the speed at the stop bar. The "v" in the all-red term is the different than that in the yellow term. The all-red "v" is the average speed of the vehicle as it traverses inside the intersection. Also "v" for the yellow term, in order to accommodate boundary conditions for allowable traffic, must be at least the speed limit. It is inappropriate for an engineer to set "v" to 5 mph slower than the speed limit, because a vehicle is *allowed* to go the speed limit and often does. By setting v lower than the speed limit, the engineer knowingly sets up a minority of law-abiding drivers to inadvertently enter the intersection illegally.

Stochastic Solutions

Stochastic solutions deal with chances, probabilities and use statistics to arrive at conclusions. As opposed to the scientific method, stochastic methods address only random events. Stochastic methods neither seek cause nor predictable patterns originating in the laws of physics because the methods assume that the events are random. Stochastic solutions are good for grading school papers but not applicable to events governed by time, distance, velocity and deceleration such as the yellow change and all-red clearance intervals. Stochastic solutions applied to non-random events at best conceal and at worst misrepresent Nature's truths.

Example 1: Ptolemy spent many nights outside counting stars. Each night Ptolemy tallied around 4000 stars and observed them traveling from east to west. Night after night for 365 days Ptolemy made his observations and recorded them. He did notice a handful of stars which did not follow the pattern of the other 4000 stars. But they were just an insignificant minority of stars. He called them "wanderers".

Ptolemy, using the stochastic method of statistics, concluded that the 99^{th} percentile of stars cross the sky each night the same way in a given year and therefore that the Earth must be at the center of the solar system.

Example 2: There are examples of stochastic yellow change interval solutions. Such are Olson and Rothery's equations, W. L. Williams' equations and the rational model equations by Fitch, Shafizadeh, Zhao, and Crowl.

Fitch's rational model makes statistical assumptions about driver behavior around dilemma zones. Fitch treats such behavior as random and like all stochastic studies, never asks what causes dilemma zones. Had Fitch used the scientific method, he would have determined that dilemma zones are not random events but are caused by the physics of the ITE yellow change interval equation. The physics of the ITE equation always establishes a "critical point" whose exact location is not known by the human driver. The unknowable always creates an indecision zone regardless of the values the engineer plugs into the equation's "constants". For turning or impeded movements, the ITE equation always forms a type I dilemma zone. The length of the dilemma zone is a function of the driver's average speed through the critical distance.

Example 3: A red light camera before-and-after study assumes that red light running events are random. Stochastic studies assume that the random behavior inherent in people is the reason behind these events, and thus adopt the common hindsight-bias that drivers are guilty. But traffic engineers know that red light running events are not random. In the very least they know that the yellow change interval significantly affects red light running.

Analytic Solutions

An analytic solution is one that usually extends a physical solution by manipulating the math. While the physical solution is true, its analytic extension may not be true.

Just because a solution is a math equation does not mean that the equation has a real-world manifestation. All physics is math, but not all math is physics. Physics is the study of the real world. Math may or may not be. (The field of physics which searches for the physical truthfulness in analytic extensions is called "mathematical physics.")

Example 1. An archer draws back his bow and shoots an arrow 500 feet. The arrow pierces a tree. A physicist knows that the arrow follows a parabolic path according to Newtonian mechanics. He derives the kinetic equation for the path. He knows that the equation is good. It is a physical solution.

Then a mathematician comes along. He extends the physicist's solution to describe the arrow's motion in reverse. The mathematician flips the sign in the equation. The new equation describes the arrow returning from the tree to the archer's bow. The mathematician believes he came up with a new revelation of Nature. The physicist, however, comes along and tells that mathematician, "Your equation does not work." The mathematician fervently defends the equation, "The math is right!"

Is the analytic solution a physical one? No. The mathematician's analytic solution does not take place in the real-world. Analytic solutions may contain some very elegant math but may not be physical solutions. In this case, the mathematician neglected the physics of entropy.

Example 2. In 1982, Kell and Fullerton (K&F) introduced the grade extension to Gazis' yellow change interval equation. K&F's extension is an analytic solution. It is not a physical one. K&F's error is that they extended the mathematics from the stopping sight distance (SSD) equation--whose mathematics applies only to emergency stopping, to the yellow change interval--whose mathematics applies to non-emergency stopping. The grade mathematics of the SSD apply only when a vehicle's maximum braking ability has been reached. Only when reached does gravity contribute to the acceleration/deceleration of the vehicle as the SSD equation describes.

8 Gates, *Dilemma Zone Driver Behavior as a Function of Vehicle Type, Time of Day and Platooning,* Transportation Research Record: Journal of the Transportation Research Board, No. 2149, Transportation Research Board of the National Academies, Washington, D.C., 2010

9 For examples, see Ceccarelli, *Uncertainty in the Yellow Change Interval*. http://redlightrobber.com.

10 Mats Järlström of Beaverton, Oregon pointed out the misapplication of the SSD grade term to the yellow change interval.

11 AASHTO Green Book, 2011, p3-3. The context is that of stopping sight distance. Therefore the context is emergencies. The context includes statements on friction which go hand in hand with emergency braking.