Determination of Left-Turn Yellow Change and Red Clearance Interval

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Abstract: The yellow change and red clearance intervals for left-turn movements in right-hand-side driving environments are addressed on an analytic basis. Factors such as safety, perception, comfort, driver's behavior, signal timing, and traffic ordinances are taken into account for setting the intervals. The yellow change interval for left-turn movements is found to be in general longer than that required for straight movements at intersections, because motorists usually slow down before making turns. Moreover, the required red clearance interval is derived in terms of the average curvature of a left-turn curve and the magnitude of acceleration that motorists are willing to bear on the curve. The calculated yellow change and red clearance intervals are in good agreement with the field observations for two signal-controlled intersections. The red clearance intervals in practice are shown to be short in comparison with the field observations and analytic estimates.

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Introduction

The signal interval between two conflict traffic movements has long been an interesting operational research problem, in which human comfort and perception factors, interpretation of the traffic ordinances, and signal timing intertwine. These factors may not be compatible with each other in practice for all situations. The setting of the yellow change and red clearance interval for straight movements has been reviewed, and explicit formulas for pretimed yellow change and red clearance intervals are discussed in detail (Liu et al. 1996). However, the setting of these signal intervals for turning movements has not been understood in both theory and practice (ITE 1985). A yellow change interval followed by a red clearance interval is utilized in practices for avoiding potential accidents among various conflict movements.

The left-turn movements may be conceptually divided into two stages. The first occurs when a motorist proceeds into an intersection during the yellow change interval, and the second takes place when the motorist makes a left turn while experiencing a relatively high magnitude of acceleration. The left-turn movement is symbolically sketched in Fig. 1 for an intersection of a simple geometry. In the figure, the longitudinal projection of a turning curve, w_l, depends on the number of lanes available in the direction of flow and the number of turning lanes. For example, if there is only one turning lane but two lanes available ahead, a motorist may turn into the inside lane while encroaching the outside lane.

The primary objective of this paper is to develop and test an analytical framework for setting yellow change and red clearance intervals for the left-turn movement, which integrates a comprehensive set of parameters related to safety, perception, human comfort, drivers' behavior, traffic ordinances, and intersection geometric characteristics. The yellow change interval for left-turn movements is found to be in general longer than that required for straight movements at intersections because motorists usually slow down before making turns. The required red clearance interval is derived in terms of the average curvature of a left-turn curve and the magnitude of acceleration which motorists are willing to bear on the curve. The field data are collected from two intersections in El Paso, Tex., to test the developed analytical framework. The next section of this paper will describe the mathematical formulation of the analytical framework. Then, examples of the field data are provided. Finally, conclusions of this research are presented.

Formulation

The time duration for completing a left turning, τ_t, may be decomposed into two parts. One is the yellow change interval y_t; the other is the red clearance interval τ_c, similar to the intervals set for straight movements, namely

$$\tau_t = y_t + \tau_c \tag{1}$$

Let us first discuss the red clearance interval. The detail of a left-turn movement can be complex. In general, the expression for a red clearance interval can be expressed as

$$\tau_c = S/\overline{v_c} \tag{2}$$

The quantity S = length of the curve measured from the stop line to L feet ahead of the clearance line; where quantity L = length of

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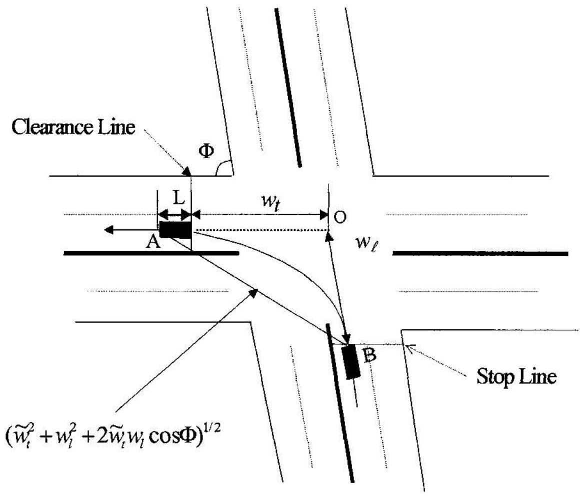

Fig. 1. Schematic drawing of left-turn movement in signal-controlled intersection

an approaching left-turn vehicle. The quantity $\overline{v_c}$ is the average speed of the vehicle on the turning curve. In order to apply Eq. (2), numerical values of the average speed $\overline{v_c}$ and the length of a curve S must be determined. Both values depend on the characteristics of the curve, which a motorist chooses. Let the turning angle be Φ (in the unit of radian), i.e., the angle shown in Fig. 1 between the direction of the approaching and that of the clearing movements. The average curvature of the turning curve is found to be Φ/S. Note that angle Φ does not have to be $\pi/2$. It can be shown that the following relation holds for the curve so long as a motorist is not making zigzag movements within the triangle BOA in Fig. 1 (need not be a right triangle):

$$S_{\min}=[\tilde{w}_t^2+w_l^2+2\tilde{w}_t w_l\cos\Phi]^{1/2}\leq S\leq S_{\max}=\tilde{w}_t+w_l \quad (3)$$

where quantity $\tilde{w}_t=w_t+L$. The trajectory of a moving vehicle at an intersection is not well defined and the detail of the maneuver is almost up to the motorist's driving habits and perception about a driving environment. Thus, the length of the turning curve may be parameterized as

$$S=\beta S_{\max}+(1-\beta)S_{\min} \quad (4)$$

where quantity $\beta\in(0,1)$. The curve could be a spiral, a circular curve, or a complex compound curve. By selecting a certain type of curve, a point of tangent (PT), and a point of curve (PC), one can estimate the exact value of parameter β. For example, choosing point B as PT or point A as PC and fitting a circular curve through point B or point A, one can find that the parameter β for this particular case is given by

$$\beta=[\Phi w_s\cot^2(\Phi/2)+|\tilde{w}_t-w_l|-S_{\min}]/(S_{\max}-S_{\min}) \quad (5)$$

where quantity $w_s=\min(\tilde{w}_t,w_l)$; and angle Φ is measured in the unit of radian. In general, a driver is willing to experience relatively high acceleration when taking parameter β as close to zero, and the driver prefers a smoother ride when taking parameter β as around 0.5.

The average curvature for the turning curve is $\kappa=\Phi/S$, and its upper bound is $\Phi/S_{\min}$. In the limit of Φ approaching zero, the movement becomes straight. As mentioned earlier, driver behavior, size of vehicles, markers for left turns, and various other factors have effects on the characteristics of left-turn curves. Nevertheless, the magnitude of average centrifugal acceleration experienced by motorists is an important indicator in determining the duration that a vehicle spends within an intersection. The detailed evaluation of acceleration and speed along a turning curve is complex. However, we may calculate the average turning speed by imposing that

$$\kappa\overline{v_c}^2\leq\gamma g \quad (6)$$

where the parameter γ may be selected in the interval [0.3, 0.8]. The number 0.3 has been selected as an "alarming" acceleration rate (Gazis et al. 1960; Stimpson et al. 1986); and the number 0.8 has been used before to represent a large deceleration rate (Hammond and Sorenson 1991). The average speed on turning curves may be set according to

$$\overline{v_c}=\min\{[\gamma g S/\Phi]^{1/2},\theta v_l+(1-\theta)v_{lt}\} \quad (7)$$

where quantities v_{lt} and v_l = speed limits along the departure and approach direction, respectively. Parameter θ, in the interval [0, 1], is to be chosen for a turning movement. If both speed limits v_{lt} and v_l are the same, the second term in the right-hand side of Eq. (7) will be independent of θ. Introducing the parameter θ is necessary for establishing a bound value for the average speed along a turning curve. For most signal-controlled intersections, the first term at the right-hand side of Eq. (7) is less than the second term (travels slowly while making a left turn). The average speed $\overline{v_c}$ along a turning curve should not exceed $\max[v_l,v_{lt}]$ for all cases according to Eq. (7), satisfying the traffic law constraints. A possible choice is to approximate θ by $(w_l/v_l)/[w_l/v_l+\tilde{w}_t/v_{lt}]$, corresponding to a situation in which a vehicle is moving at speed limit along both sides BO and OA shown in Fig. 1.

The time duration needed for a motorist to clear off an intersection may be estimated using Eqs. (2)–(4) as

$$\tau_c\cong[\beta S_{\max}+(1-\beta)S_{\min}]/\overline{v_c} \quad (8)$$

The upper bound given by Eq. (8) is usually larger than the red clearance interval for straight movement for most intersections, if not for all (Liu et al. 1996).

Next, the yellow change interval needed is discussed for a vehicle approaching a signalized intersection at a speed v_0. The point of no return for such a vehicle is the distance (measured from the stop line) beyond which a motorist cannot stop comfortably and safely. It is given (Gazis et al. 1960) by

$$x_a=v_0\delta_-+v_0^2/2a_- \quad (9)$$

where a motorist's perception-reaction time and the comfortable deceleration rate for aborting the movement are represented by δ_- and a_-, respectively. When making a turn, a motorist can either decelerate or accelerate toward the intended entering speed v_i, less than or equal to the speed limit v_l. Speed v_i may be expressed in terms of the speed limit and the average turning speed, i.e.

$$v_i=\begin{cases}\alpha v_l+(1-\alpha)\overline{v_c} & \text{if } \overline{v_c}\leq v_l\\ v_l & \text{otherwise}\end{cases} \quad (10)$$

where quantity α is a parameter. It would be difficult to pinpoint an exact value of parameter α because of its dependence on driver's behavior and decisions. The magnitude of acceleration that a driver is willing to experience on curves increases with parameter α. Parameter α need not be the same for different signalized intersections. When speed $\overline{v_c}$ is equal to the approaching speed limit v_l, the entering speed is equal to the speed limit, indepen-

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dent of the parameter α. The entering speed may be reduced along a turning curve, indicating that the average speed can be substantially less than the entering speed.

If a vehicle approaching an intersection is of little or no acceleration power or is moving at a constant speed, the yellow time required for motorists proceeding into the intersection legally beyond the point of no return (Gazis et al. 1960) is

$$y_0 = \delta_- + v_0/2a_- \tag{11}$$

Assuming that the deceleration and acceleration toward the speed v_i can be achieved before the vehicle crosses the stop line, the yellow duration needed is given by

$$y_a = 2y_0v_0/(v_0+v_i) = 2(\delta_- + v_0/2a_-)/(1+v_i/v_0) \tag{12}$$

The yellow change interval calculated using Eq. (12) is longer (shorter) than that estimated using Eq. (11) if a vehicle is approaching an intersection at a speed higher (lower) than the entering speed v_i. The yellow change interval that is good for all vehicles proceeding into an intersection legally is determined by Eq. (11) when $v_0 = v_i$, yielding

$$y_t = 2(\delta_- + v_l/2a_-)/(1+v_i/v_l) \tag{13}$$

When the entering speed increases to the speed limit, Eq. (13) gives the same result as that for straight movement (Liu et al. 1996). This corresponds to a situation where a driver is not slowing down before making a turn. The total time interval needed for completing an entire turning process is found by summing Eqs. (8) and (13)

$$\tau_t = 2(\delta_- + v_l/2a_-)/(1+v_l^{-1}v_i) + [\beta S_{\max} + (1-\beta)S_{\min}]/\overline{v_c} \tag{14}$$

The three parameters α, β, and γ associated with Eq. (14) can be adjusted to fit driving conditions at intersections. The main factors, such as posted speed limits, geometric features of intersections, driver's comfort on straight and curved movements, and driver's perception-reaction time, are integrated together in Eq. (14).

One may seek some deduction on the clearance interval by considering the distance between the potential conflicting points of the turning movements and the stop lines of the other conflicting movements. In other words, if this distance is big, the chance to have a collision is small. Such a deduction is suggested by Williams (1977). However, one should be careful in making the deductions for different conflict movements by considering all possible scenarios in which accidental collisions might occur. In general, the deduction in clearance interval is small and is on the order of 0.1 s. For large intersections with a width of 50 m or more (extreme case), the deduction can go higher depending on the geometry, the signal control method, and the speed limits in various conflict movements at the intersections. We do not suggest shortening the clearance interval because the deduction of clearance interval is case dependent and its magnitude is small. If insisting on deducting the clearance interval, one should make sure that the deduction is appropriate based on observational and/or realistic simulations.

Examples

This research attempted to collect the field data for the left-turn movements from two signal-controlled intersections at El Paso, Tex., in order to test the analytical framework developed in the last section. The times spent by the last vehicle (in one signal cycle) within an intersection (going beyond the clearance line)

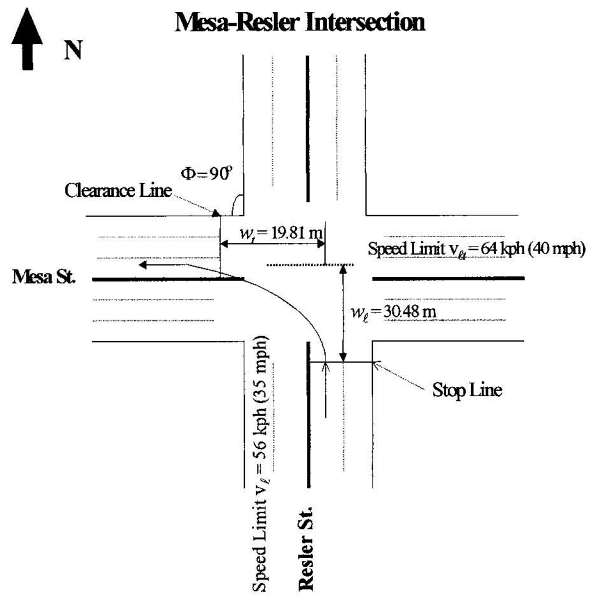

Fig. 2. Geometric layout of Mesa-Resler intersection in El Paso, Tex.

after onset of the red clearance interval are collected for the Mesa-Resler (M-R) and Airway-Montana (A-M) intersections in El Paso, Tex., using stopwatches. Figs. 2 and 3 illustrate geometric layouts of two intersections. The intersecting angle Φ are estimated to be $\pi/2$ for the M-R intersection and 1.66 rad for the A-M intersection, respectively. The left turns observed in the M-R intersection are made by motorists heading north on Resler Dr., and those observed in the A-M intersection are for vehicles heading north on Airway Blvd. A total of 159 observations are recorded for the M-R intersection and a total of 141 are recorded for the A-M intersection. The actual yellow change and red clearance interval settings at the two intersections are also recorded.

The widths w_l and w_t for the M-R (A-M) intersection are found to be 30.48 m (20.42 m) and 19.81 m (14.02 m), respec-

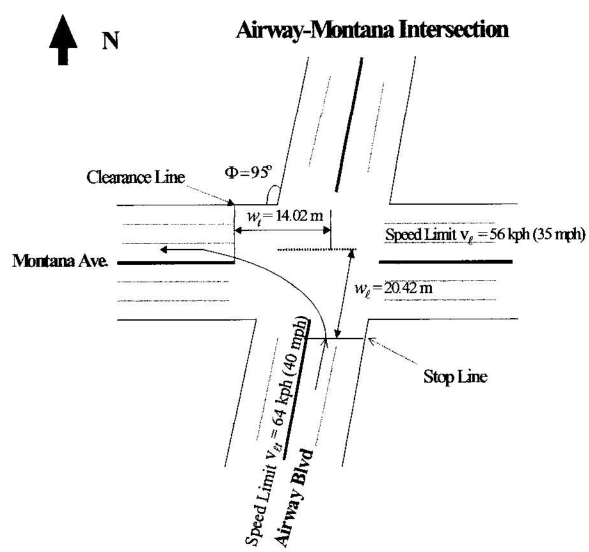

Fig. 3. Geometric layout of Airway-Montana intersection in El Paso, Tex.

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Table 1. Basic Parameters for Mesa-Resler Intersection

Parameter	Value
w_l	30.48 m
w_t	19.81 m
L	4.06 m
v_l	15.56 m/s
Φ	1.57
$\tilde{w}_t$	23.87 m
g	9.81 m/s^2
v_{lt}	17.78 m/s
$S_{\min}$	38.71 m
$S_{\max}$	54.35 m
Θ	0.60
$\Theta v_i+(1-\Theta)v_{lt}$	16.45 m/s

tively. The posted speed limit is 15.56 m/s (35 mi/h) for both Resler Dr. and Montana Ave. and is 17.78 m/s (40 mi/h) for both Mesa St. and Airway Blvd. The length of the vehicle L is set to 4.0 m. These parameters as well as other parameters of the two intersections for calculating the red clearance interval are shown in Tables 1 and 2.

By setting the parameters β and γ and using Eq. (8), the required red clearance interval τ_c can be computed. Fig. 4 illustrates curves of calculated red clearance interval versus γ for different values of β, and Fig. 5 shows curves of red clearance interval versus β for different values of γ for the M-R intersection. Similar graphs are also plotted for the A-M intersection, as shown in Figs. 6 and 7. Different red clearance intervals can be obtained by adjusting parameters β and γ. Parameter γ usually falls into the range between 0.3 and 0.8, as indicated earlier, but curves associated with values of γ beyond this range are also plotted in the figures.

Figs. 8 and 9 illustrate cumulative percentages of vehicle clearance time distributions for the two intersections. As shown in the figures, the maximum clearance time is 3.8 s for the M-R intersection and 3.0 s for the A-M intersection. These required clearance times can be matched by the calculated red clearance intervals with several possible combinations of β and γ values, as shown in Figs. 4–7. Since the red clearance interval is more sensitive to γ than β, we will determine a numerical value of parameter β first. With a reasonable assumption of a circular curve, the exact β value can be calculated using Eq. (5), and the result is 0.35 for the M-R intersection and 0.30 for the A-M intersection. With these β values, it is found that $\gamma=0.50$ results in a red clearance interval of 3.8 s for the M-R intersection and $\gamma=0.55$ results a red clearance interval of 3.0 s for the A-M intersection. These γ values are selected such that those vehicles entering the intersection when the yellow interval terminates and following a circular curve path will have enough time to clear the intersection before the clearance interval ends. Although there are not many such vehicles in the field, as shown by Figs. 8 and 9, they do need to be considered in the determination of red clearance intervals for safety reasons. Therefore, a combination of $\beta=0.30\sim0.35$ and $\gamma=0.50\sim0.55$ may be used as default values in setting the red clearance interval in practice if no other relevant information is available. It is found that the observed clearance time is considerably longer than the actual setting of the interval,

Table 2. Basic Parameters for Airway-Montana Intersection

Parameter	Value
w_l	20.42 m
w_t	14.02 m
L	4.06 m
v_l	17.78 m/s
Φ	1.66
$\tilde{w}_t$	18.08 m
g	9.81 m/s^2
v_{lt}	15.56 m/s
$S_{\min}$	26.07 m
$S_{\max}$	38.50 m
Θ	0.52
$\Theta v_i+(1-\Theta)v_{lt}$	16.71 m/s

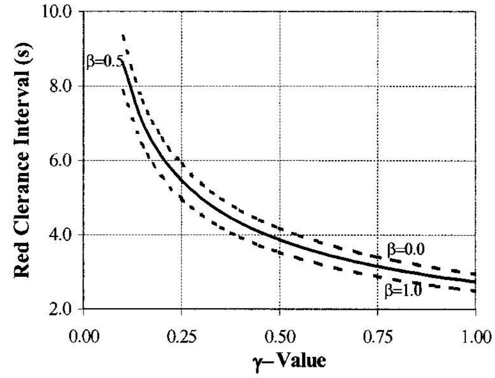

Fig. 4. Curves of calculated red clearance interval versus γ for different β values for Mesa-Resler intersection

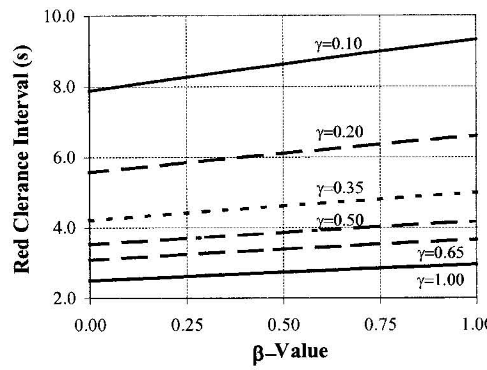

Fig. 5. Curves of calculated red clearance interval versus β for different γ values for Mesa-Resler intersection

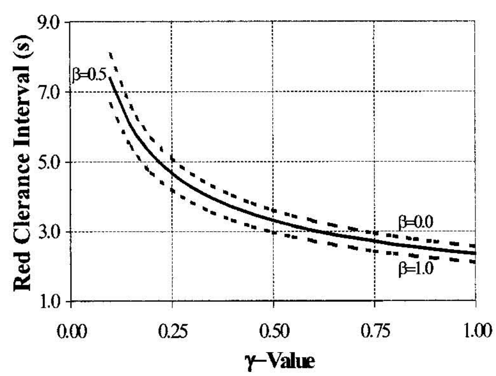

Fig. 6. Curves of calculated red clearance interval versus γ for different β values for Airway-Montana intersection

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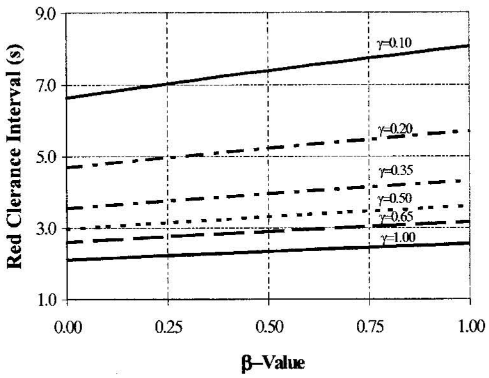

Fig. 7. Curves of calculated red clearance interval versus β for different γ values for Airway-Montana intersection

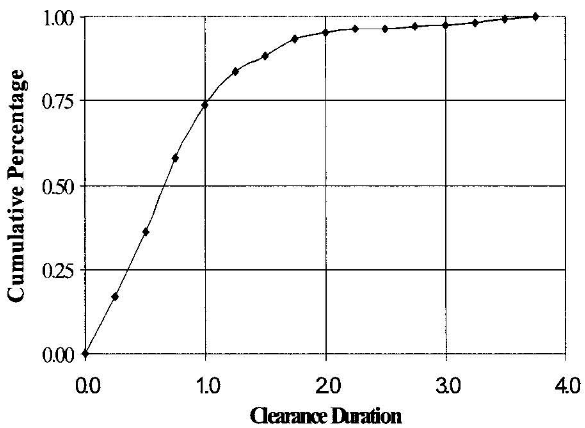

Fig. 8. Cumulative plot of collected red clearance intervals for vehicles entering intersection M-R before yellow duration ends to clear intersection

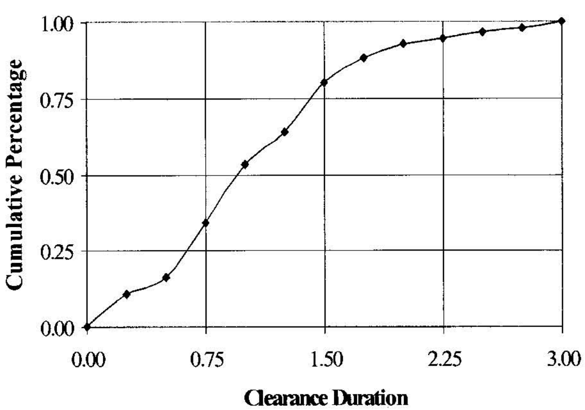

Fig. 9. Cumulative plot of collected red clearance intervals for vehicles entering intersection A-M before yellow duration ends to clear intersection

which is 1.0 s for the two examined intersections. The continuous use of this short red clearance interval may impose danger to vehicles that enter the intersections late but legally.

The yellow change interval depending on the entering speed is given by Eq. (13), and the entering speed can be determined by varying parameter α in Eq. (10). Setting γ to 0.55 and choosing parameter α to be 1.0, 0.5, and 0, the yellow change intervals for the A-M intersection are found to be 4.0, 4.5, and 5.1 s, respectively. Repeating the calculation but setting γ to 0.5 for the M-R intersection, the required yellow change intervals are found to be 3.6, 3.8, and 4.1 s. In the above calculation, the perception-reaction time δ_- and the comfortable deceleration rate are set to a_- to 1.0 s and 3.0 m/s^2 (ITE 1985; Kell and Fullerton, 1991). The actual setting of the yellow duration is 4.0 s for both intersections. For the M-R intersection, the yellow time is long enough even for vehicles slowing down to an entering speed that is close to the average speed on a turning curve. However, the yellow change interval appears to be short for vehicles entering the A-M intersection with a speed lower than the speed limit. Those drivers at the far end of the dilemma zone when the yellow light commences must rush into the A-M intersection at a speed close to speed limit in order to beat the yellow time. The tradeoff is that they must bear a relatively high acceleration/deceleration when clearing the intersection within the red clearance interval. For facilitating turning movements, the default value for parameter α might be set to 0.5 in the determination of the yellow change interval. Smaller α values may be used if a longer yellow change interval is suggested by further experimental observations.

It should be noted that the above discussions are based only on the two intersections from which the field data were collected. It is suggested that further efforts be made to collect field data from intersections with diverse geographical conditions.

Conclusion

An analytic framework, dealing with both the yellow change and the red clearance interval for left-turn movements (in a right-hand-side driving environment) in signal controlled intersections, is presented by integrating coherently the human comfort, perception, safety, and traffic ordinances. An entire turning process may be decomposed into two steps. The first step is to decelerate or accelerate the vehicle toward the intersection in order to proceed into a turning curve. The second is to clear the intersection. Explicit formulas are derived for the required time duration for completing the steps. Three important parameters, α, β, and γ, are introduced to characterize the turning process. The parameter α is introduced to describe the acceleration/deceleration action made by a driver right before making turns; parameter β characterizes the length of the turning curves; and the parameter γ represents human comfort factor for turning movements. Parameter θ and intersecting angle Φ are introduced to reflect the traffic rules imposed on turning movements and to characterize the physical layout of signal-controlled intersections, respectively.

The estimated yellow change and red clearance intervals are compared with field settings. It is found that the yellow duration provided in the M-R intersection is appropriate but the one for the A-M intersection may not be long enough for drivers making a comfortable left turn. Moreover, it appears that the red clearance setting in practice is shorter than what actual observations suggest if the starting lost time of vehicles in the conflicting approach is not considered. However, the observed clearance intervals agree well with the presented analytic estimates with parameters $\beta = 0.30-0.35$ and $\gamma = 0.50-0.55$. Our studies suggest that both the yellow change and the red clearance interval for left turn movements should be set in general longer than that for a straight movement.

Although the yellow change and red clearance intervals for straight movements are discussed in practical handbooks, such as in the *Manual of traffic signal design* (Kell and Fullerton 1991),

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estimations for these intervals for turning movements have not been given and put into practice before. The red clearance interval in practices is usually set within the interval of [0.5 s, 2.0 s]. As can be seen from the presented examples, a 2 s clearance interval may not be long enough for making a complete turn. On the other hand, a short red clearance interval seems to work satisfactorily for some intersections. The success in applying a short clearance interval relies on the following actions:

- The last vehicle makes a turn sometime before a yellow interval terminates,
- The last vehicle entering into an intersection late but legally will clear the intersection swiftly, or
- Vehicles at conflict approaches yield to a left-turn vehicle after a short clearance interval is over (starting lost time).

This chain of actions may fail under inclement weather conditions when the road surface becomes slippery and/or visibility becomes low. It is not difficult to conceive a situation in which a potential accident may occur.

The numerical values for parameters α, β, and γ to be used for determining both the yellow change and red clearance intervals expressed in Eq. (14) should be set to approximately 0.5, 0.35, and 0.55 as default values according to our observational results; however, these values may be modified to fit the observations for different intersections. Note that not all yellow change intervals at controlled intersections set for left-turn movements are appropriate. Parameters α, β, and γ are related to driver behavior at controlled intersections and therefore may vary to some extent over different geographic regions. Therefore, it is suggested that data be collected widely from more intersections in order to determine the average values or the site-specific values of α, β, and γ. One way to determine these parameters for a given intersection is to videotape the vehicle left-turn movements. From the video images, one can estimate for each left-turn movement the entering speed, the parameter β, the time spent by a vehicle within the intersection, and the average speed of a vehicle within an intersection. Then, the distribution of the parameters α and γ can be assessed using Eqs. (10) and (6), respectively. Note that there are three numerical values α, β, and γ associated with each left turn. Using the distributions of the parameters α, β, and γ, a traffic engineer should be able to find a reasonable red clearance interval, which is expected to be consistent with drivers' behavior in making left turns. The results obtained in this paper can be applied equally well to right-turn movements if left-hand-side driving convention is practiced in a driving environment.

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Notation

The following symbols are used in this paper:

a_- = comfortable deceleration rate when approaching controlled intersection (m/s^2);
g = gravity acceleration rate on Earth (m/s^2);
L = vehicle length (m);
S = length of turning curve for left-turn movements (m);
$S_{\min}$ = minimum length of turning curve for left-turn movements (m);
$\bar{v}_c$ = average speed of vehicle on turning curve (m/s);
$\tilde{v}_c$ = maximum average speed of vehicle on turning curve (m/s);
v_i = vehicle speed when entering intersection for making left turn (m/s);
v_l = speed limit along approaching direction before making left turn (m/s);
v_{lt} = speed limit along departure direction (m/s) when finishing left turn (m/s);
v_0 = approaching speed of vehicle toward intersection when yellow interval commences (m/s);
w_l = projection length of turning curve along approaching direction for left-turn movements (m);
w_t = projection length of turning curve along departure direction for left-turn movements (m);
$\tilde{w}_t$ = summation of width w_t and vehicle length L (m);
x_a = deceleration distance needed for stopping vehicle approaching intersection (m);
y_t = yellow interval for left-turn movements (s);
α = dimensionless parameter ranging from 0 to 1 for estimating speed v_i;
β = dimensionless parameter ranging from 0 to 1 for estimating turning length S;
γ = dimensionless parameter ranging from about 0.3 to 0.8 for estimating magnitude of acceleration rate that driver is willing to bear on turning curves;
δ_- = perception-reaction time for decelerating vehicle (s);
θ = dimensionless parameter ranging from 0 to 1 for estimating turning speed;
τ_t = summation of yellow interval and red clearance interval for left-turn movements (s);
τ_c = red clearance interval for left-turn movements (s); and
Φ = intersecting angle between vehicle approach and departure direction (radians).

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